Complementary Monopolies and Bargaining

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Abstract

How should complementarities affect antitrust merger policy? I introduce a two-stage strategic model in which complementary-input monopolists offer supply schedules to producers and then engage in bilateral bargaining with producers. The main result is that there is a unique weakly dominant strategy equilibrium, the equilibrium attains the joint-profit-maximizing outcome, and output equals that of a bundling monopoly. The result holds with perfect competition in the downstream market with both unit capacity and multiunit capacity. The result also holds with oligopoly competition in the downstream market. The result contrasts with the Cournot effect, which states that complementary-input monopolists choose total prices that are greater than the bundled-monopoly level. The analysis shows that consumers' surplus and total producers' surplus are greater with supply schedules and bargaining than with posted-price competition. The analysis has implications for antitrust policy toward vertical and conglomerate mergers.

1. Introduction

Complementarities have been used to seek antitrust approval of conglomerate and vertical mergers, as in the blocked merger of General Electric (GE) and Honeywell. To examine the economic effects of complementarities, I introduce a two-stage bargaining game that provides a more complete description of interaction between complementary monopolists and downstream producers. In the first stage of the game, each complementary monopolist offers an input supply schedule to downstream producers. Then, in the second stage of the game, each complementary monopolist engages in Nash-in-Nash bargaining with each producer over input prices. Given these supply schedules and input prices, producers choose input demands and supply final outputs, and the downstream market clears. At the unique weakly dominant strategy equilibrium, industry output attains the joint-profit maximum, and total input prices are less than the monopoly

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markup over producers' marginal cost. The efficiency of the equilibrium outcome has implications for antitrust policy toward vertical and conglomerate mergers.

The two-stage bargaining game describes markets in which firms negotiate supply contracts. For many markets, contract negotiation offers a more accurate description of business transactions than does the basic posted-price model. Companies use supply contracts because business transactions often take place over time and require capacity commitments from suppliers and demand commitments from buyers. Industries often use contracts for supply-chain management and coordination (see Tsay 1999; Tsay, Nahmias, and Agrawal 1999; Cachon and Lariviere 2005; Li and Wang 2007; Arshinder, Kanda, and Deshmukh 2011). There is extensive evidence that suppliers negotiate supply contracts with producers, assemblers, and distributors. According to the Bureau of Labor Statistics (BLS), US companies have over 1,740,000 wholesale and manufacturing sales representatives.[1] The 2015 BLS data also show that US companies have over 72,000 purchasing managers and over 400,000 buyers and purchasing agents who evaluate suppliers, review product quality, and negotiate supply contracts.[2]

The main results of the analysis are as follows. First, I consider the two-stage game when the downstream market is perfectly competitive and producers have unit capacity. I show that the strategic game in supply schedules has a unique weakly dominant strategy equilibrium. I find that at the unique equilibrium of the strategic game, suppliers and producers maximize joint benefits. Industry output downstream equals the cooperative level, and total input prices are strictly less than the monopoly markup of the final price over producers' unit costs. The analysis suggests that complementarity of inputs induces coordination instead of blocking it.

The intuition for the efficiency result is as follows. The game is solved by backward induction. In the second stage of the game, simultaneous bilateral bargain-

[1] The Bureau of Labor Statistics (BLS) data are for 2015 and include the occupational categories 41-4011 (BLS, Occupational Employment Statistics: 41-4011 Sales Representatives, Wholesale and Manufacturing, Technical and Scientific Products [http://www.bls.gov/oes/current/oes414011.htm]) and 41-4012 (BLS, Occupational Employment Statistics: 41-4012 Sales Representatives, Wholesale and Manufacturing, Except Technical and Scientific Products [http://www.bls.gov/oes/current/oes414012.htm]).

[2] Purchasing managers are in category 11-3061 (BLS, Occupational Employment Statistics: 11-3061 Purchasing Managers [https://www.bls.gov/oes/current/oes113061.htm]). Buyers and purchasing agents are in the categories 13-1022 (BLS, Occupational Employment Statistics: 13-1022 Wholesale and Retail Buyers, Except Farm Products [http://www.bls.gov/oes/current/oes131022.htm]) and 13-1023 (BLS, Occupational Employment Statistics: 13-1023 Purchasing Agents, Except Wholesale, Retail, and Farm Products [http://www.bls.gov/oes/current/oes131023.htm]). The BLS states that purchasing managers "[p]lan, direct, or coordinate the activities of buyers, purchasing officers, and related workers involved in purchasing materials, products, and services. [The group] [i]ncludes wholesale or retail trade merchandising managers and procurement managers" (see BLS, Occupational Employment Statistics: 11-3061 Purchasing Managers [http://www.bls.gov/oes/current/oes113061.htm]). The BLS also states that "[p]urchasing agents and buyers consider price, quality, availability, reliability, and technical support when choosing suppliers and merchandise." "Buyers and purchasing agents buy products and services for organizations to use or resell. They evaluate suppliers, negotiate contracts, and review the quality of products" (BLS, What Buyers and Purchasers Do [http://www.bls.gov/ooh/business-and-financial/buyers-and-purchasing-agents.htm#tab-2]).

ing over the division of economic rents provides incentives for entry of downstream producers. Competitive entry in the downstream market generates an output equal to the smallest of the maximum input supply offers. In addition, as a consequence of bilateral bargaining, input suppliers obtain shares of total returns. In the first stage of the game, complementarity implies that input suppliers take into account the potential effects of their supply decisions on the product market. If other input suppliers were to choose maximum quantities above those that maximize joint benefits, then a supplier would strictly prefer to offer a lower maximum quantity that would maximize joint benefits. If other input suppliers were to choose maximum quantities below those that maximize joint benefits, then a supplier would not restrict the quantity further and would be indifferent between all maximum quantities above the level that maximizes joint benefits. So the maximum quantity that maximizes joint benefits is the unique weakly dominant strategy for every supplier.

Second, I show that the outcome of the two-stage game generates greater consumers' surplus, total producers' surplus, and social welfare than the Cournot posted-price game. With posted prices, complementary monopolists behave inefficiently because they do not consider how their prices affect each other's profits, which generates a free-rider problem. This leads to the Cournot effect: competing complementary monopolists choose higher total input prices than the cooperative outcome.[3] The present analysis shows that bargaining over supply schedules eliminates the Cournot effect. Economists have applied the Cournot effect to many problems, including vertical and conglomerate mergers, bilateral monopoly, successive monopolies, negotiations between labor and management, international trade, money in decentralized exchange, externalities, joint production, innovation, and coordination in network industries. Despite the wide application of the Cournot effect, the stark contrast between cooperation and competition may be due to artificially restricting competition to posted prices.

Third, I extend the two-stage bargaining game to consider a perfectly competitive downstream market in which producers have multiunit capacity and input suppliers have the option of offering two-part tariffs to producers. In the first stage, input suppliers offer aggregate supply schedules and supply contracts to individual producers. In the second stage, input suppliers and producers bargain over transfers. I show that there exists a unique weakly dominant strategy equilibrium in supply schedules and supply contracts, and equilibrium transfers are unique. Because inputs are complements, aggregate input supplies and the number of supply contracts maximize the joint benefits of suppliers and producers. In addition, producers operate at minimum efficient scale. Total payments to input suppliers are strictly less than each producer's revenues net of production costs at the bundled-monopoly output.

[3] According to Cournot (1897, p. 103), "An association of monopolists, working for their own interest, in this instance will also work for the interest of consumers, which is exactly the opposite of what happens with competing producers." Cournot finds that "the composite commodity will always be made more expensive, by reason of separation of interests than by reason of the fusion of monopolies." See also Moore (1906).

Fourth, I extend the complementary-monopolies Cournot model to allow input suppliers to offer two-part tariffs when downstream producers have multiunit capacity. This generalizes the literature on competition with two-part tariffs to complementary goods.[4] I show that input suppliers offer only per-unit tariffs and that the Cournot effect continues to hold with posted prices. I then compare the two-stage bargaining game with the posted-price game when producers have multiunit capacity. I find that output in the two-stage bargaining game is the same as for a bundled monopoly with two-part tariffs. This implies that the two-stage bargaining game generates greater industry output than the Cournot posted-price equilibrium. The two-stage bargaining game, in comparison with posted prices, also increases consumers' surplus, total producers' surplus, and social welfare.

Fifth, I extend the two-stage bargaining game to oligopoly competition among producers in the downstream market. I show that the strategic game in supply schedules has a unique weakly dominant strategy equilibrium. Again, I find that at the unique equilibrium of the strategic game, suppliers and producers maximize joint benefits. The downstream industry output equals the output when inputs are supplied by a bundled monopoly and total payments are strictly less than the markup over producers' costs times output per producer.

Finally, I explore the implications of the results for antitrust policy in markets with complementary inputs or complementary final products. The main implication of the results is that vertical or conglomerate mergers are not necessary for markets to achieve a cooperative outcome. This means that the Cournot effect need not justify mergers unless it can be established that firms engage in posted-price behavior instead of forming supply contracts. So vertical and conglomerate mergers need not improve market outcomes. I consider antitrust policy toward conglomerate mergers as in the blocked GE-Honeywell merger. I also consider mergers of complementary suppliers that would generate bilateral monopoly with a bundled input supplier and a monopoly downstream firm. Applying the present two-stage bargaining model, I examine antitrust policy toward vertical mergers of multiple successive monopolies and again show that mergers need not improve market outcomes.

In the second stage of the game, each input supplier bargains bilaterally with each downstream producer over the input price. Bilateral bargaining follows the Nash-in-Nash bargaining solution, with each bargaining pair taking the equilib-

[4] It is well known that an upstream monopolist serving a downstream monopolist can achieve the joint-profit maximum with two-part tariffs. In addition, a monopolist serving a competitive downstream industry can achieve efficiency because it can determine the final price and extract profits by using two-part tariffs (see Coase 1946; O'Brien and Shaffer 1992). Two-part tariffs do not achieve the cooperative outcome when there is competition upstream among sellers offering substitutes; see Rey and Stiglitz (1988) on two-part tariffs and Spulber (1989) on nonlinear pricing. Two-part tariffs also do not achieve the cooperative outcome when there is imperfect competition downstream; see Mathewson and Winter (1984) on two-part tariffs in the context of vertical restraints. Two-part tariffs offered by suppliers of substitute or complementary goods need not be unique with a single downstream buyer and need not be efficient when buyers are heterogeneous (Calem and Spulber 1984).

rium outcome of other bargains as given (see Harsanyi 1959, 1963). For an overview of Nash-in-Nash bargaining and an extension to noncooperative strategies, see Collard-Wexler, Gowrisankaran, and Lee (2016). The basic Nash bargaining solution is described in Nash (1950, 1953), Harsanyi and Selten (1972), Roth (1979), and Binmore (1987). The basic Nash cooperative bargaining strategy has been extended to noncooperative strategies; see Rubinstein (1982) and Binmore, Rubinstein, and Wolinsky (1986). Empirical studies examine the implications of Nash-in-Nash bargaining in a variety of industries; see the discussion in Collard-Wexler, Gowrisankaran, and Lee (2016) and the references therein.

The present analysis suggests that allowing for more general strategic interactions is sufficient to resolve the complementary-monopolies question. The Cournot effect has generated nearly 2 centuries of controversy involving many distinguished economists.[5] Some economists argue that market outcomes are inefficient as predicted by the Cournot effect, and other economists argue that cooperative bargaining among complementary monopolies would result in an efficient outcome (see Bowley 1928; Wicksell [1934] 2007; Tintner 1939; Henderson 1940; Leontief 1946; Fellner 1947).[6] Schumpeter (1928) suggests that Cournot duopolists (or complementary monopolists) would maximize joint profits through tacit coordination.[7] The present discussion differs from the traditional literature in that complementary monopolists bargain with producers rather than with each other.

There is a long literature on Cournot's complementary-monopolies problem and its dual, the quantity-competition model.[8] Edgeworth (1925) considers competition with imperfect complements and substitutes and points out that perfect complementarity is a limiting case of complementary goods. Economides and Salop (1992) and Denicolo (2000) consider complementarities in consumption.

[5] Works that have considered Cournot's analysis include Fisher (1898), Moore (1906), Marshall (1907), Bowley (1924), Edgeworth (1925), Schumpeter (1928), Zeuthen (1930), Stackelberg (1934), Hicks (1935), Kaldor (1936), and Tintner (1939). Machlup and Taber (1960) provide a valuable overview of the early literature.

[6] For example, Bowley (1928, pp. 656–57) considers a bilateral monopoly in which "the manufacturer and supplier of material combine to maximise their joint gain" and points out that the same result is obtained "when the manufacturer uses a number of materials, each the subject of an independent monopoly." Bowley expresses concern that the bargaining outcome is "unstable" because each side may want a larger share of the total benefit. Machlup and Taber (1960, p. 111) note that "negotiations between separate monopolists would, in the case of intermediate products, necessarily be carried on in terms of both quantity and price, and that the quantity agreed upon between the parties would be the same as that produced by an integrated monopolist."

[7] Schumpeter (1928, p. 370) states, "[W]e are, first, faced by the fact that they cannot very well fail to realise their situation. But then it follows that they will hit upon, and adhere to, the price which maximises monopoly revenue for both taken together (as, whatever the price is, they would, in the absence of any preference of consumers for either of them, have to share equally what monopoly revenue there is). The case will not differ from the case of conscious combination—in principle—and be just as determinate."

[8] Edgeworth (1925) critiques the stability of the Cournot duopoly models for both substitutes and complements, and Fisher (1898, pp. 126–28) critiques the dynamic analysis in Cournot's basic duopoly models. Works that consider the effects of conjectural variations on Cournot duopoly include Frisch (1951) and Hicks (1935). Stackelberg (1934) considers Cournot reactions in successive moves.

Singh and Vives (1984) compare quantity and price strategies in a one-stage game with differentiated products that are either imperfect complements or substitutes.[9] It can be shown that as products approach perfect complementarity, the quantity-setting equilibrium with complementarity in Singh and Vives (1984) approaches the monopoly outcome.

The economics literature provides many examples of complementary monopolies, including copper and zinc monopolists selling to downstream producers of brass (Cournot 1897), railroad lines (Ellet [1839] 1966, pp. 77–78), and links in a chain of canals (Edgeworth 1925, p. 124). Choi (2008) discusses the complementarity between inputs such as jet engines and avionics in aircraft component markets. Denicolo (2000) considers markets with generalist and specialist firms that respectively produce all or some of the complements in the market, including, for example, color film and photo finishing. Casadesus-Masanell and Yoffie (2007) develop a dynamic pricing version of Cournot's complements model and study competition between Microsoft's Windows operating system and Intel's microprocessors. Laussel (2008) examines Nash bargaining over prices of complementary components in automobiles and aircraft. Laussel and Van Long (2012) extend Laussel (2008) with a dynamic equilibrium analysis of the downstream firm's divestiture of complementary suppliers. Llanes and Poblete (2014) examine ex ante agreements with complementarities and technology standard setting. Spulber (2016) examines patent licensing with innovative complements and substitutes and considers the effects of competitive constraints.

On the properties of games with general complementary strategies, see Topkis (1998) and Vives (2000, 2005). Legros and Matthews (1993) show that there is an efficient Nash equilibrium in a partnership with strictly complementary efforts, although in their setting there is a continuum of Nash equilibria without this property. Hirshleifer (1983, 1985) considers complementary efforts in a public-goods model with a continuum of Nash equilibria.

The present two-stage bargaining model with supply schedules and Nash-in-Nash bargaining over prices differs from Cournot's one-stage game with posted prices. The present model also differs from Cournot's one-stage quantity-competition model in which products are perfect substitutes. The present model further differs from Bertrand's (1883) one-stage model of price setting in which goods are perfect substitutes. In Bertrand's model, prices fall to players' marginal costs, whereas in the present model with supply schedules, all players choose quantities equal to the monopoly outcome.

The discussion is organized as follows. Section 2 presents the two-stage model of complementary monopolies and characterizes the equilibrium when the downstream market is perfectly competitive. Section 3 extends the model with perfect competition downstream to allow multiunit capacity and two-part tariffs. Section 4 considers complementary monopolies when the downstream mar-

[9] Singh and Vives (1984, p. 553) observe that "Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. Exchanging prices and quantities, we go from one to the other." See also Vives (1985).

ket has oligopoly competition with differentiated products. Section 5 discusses antitrust-policy implications of the analysis, including conglomerate and vertical mergers, successive monopoly, and bilateral monopoly. Section 6 concludes the discussion. The proofs of the main propositions appear in the Appendix.

2. Complementary Monopolies with Perfect Competition in the Downstream Market

This section introduces a two-stage game with complementary monopolists that supply inputs to perfectly competitive downstream producers. In the first stage, input suppliers choose binding supply offers noncooperatively, and entry of producers determines the demand for inputs. In the second stage, each input supplier bargains with producers over input prices, and the input and output markets clear.

2.1. Producers

The downstream market is perfectly competitive with a homogeneous final good as in Cournot's model. Let p denote the price of the final good, and let q be the output of the downstream industry. Assume that the market demand $q = D(p)$ is strictly decreasing and twice continuously differentiable. Let $p = P(q)$ denote the market inverse demand.

Each producer has unit capacity. This restriction is for ease of presentation only. Section 3 considers downstream competition when producers have multi-unit capacity and shows that the results continue to hold. In subsequent sections, I also consider downstream oligopoly and monopoly when producers have multi-unit capacity.

There are n inputs that are perfect complements as in Cournot's model.[10] Each producer's costs consist of a unit cost c and the sum of the purchase prices of the complementary inputs. Prices $r_1, r_2, \ldots, r_n$ differ across inputs. When the industry output is q, each producer earns a profit of

$$\Pi(q, r_1, r_2, \ldots, r_n) = P(q) - c - \sum_{i=1}^{n} r_i. \tag{1}$$

Producers are active if and only if $\Pi(q, r_1, r_2, \ldots, r_n) \geq 0$.

2.2. Input Suppliers

In the first stage of the game, each input supplier i makes a binding commitment to provide whatever quantity q of its input that producers demand up to a maximum amount y_i. Each input supplier offers a supply schedule $Y_i(q)$ given by

$$Y_i(q) = \min\{q, y_i\} \qquad (i = 1, \ldots, n). \tag{2}$$

[10] With unit capacity, each producer's technology can be represented by a Leontief production function $x = \min\{\delta_1, \delta_2, \ldots, \delta_n\}$, where x is the producer's output, δ_i equals one if the producer uses input i and zero otherwise.

To simplify notation, let the maximum levels $y_1, y_2, \ldots, y_n$ represent the supply offers $Y_1(q), Y_2(q), \ldots, Y_n(q)$. When choosing their supply schedule offers, input suppliers do not know the supply offers of other input suppliers, nor do they know the amount q that will be demanded by producers. I consider weakly dominant strategy equilibria in supply offers.

Because inputs are perfect complements, downstream output is bounded by the smallest of the maximum input supply offers: $q \leq y_{\min}$, where $y_{\min} \equiv \min\{y_1, y_2, \ldots, y_n\}$. Assume that downstream producers enter the market sequentially so that each active producer is able to obtain all of the inputs until output reaches $y_{\min}$. Bargaining in the second stage implies that all active producers earn nonnegative profits. Entry of downstream producers continues until total demand for inputs equals the minimum of the maximum input supply offers,

$$q = y_{\min}. \tag{3}$$

As in Cournot's complementary-monopolies model, input suppliers produce to order instead of producing to stock.[11] Each input supplier i incurs costs $k_i q$ ($i = 1, 2, \ldots, n$) on the basis of the amount of the input that is demanded by producers. Because prices are symmetric and given input demand q, each input supplier i earns profits

$$V_i(q, r_i) = (r_i - k_i)q \qquad (i = 1, 2, \ldots, n). \tag{4}$$

Input suppliers are active if and only if $V_i(q, r_i) \geq 0$.

In the second stage of the game, each input supplier i bargains bilaterally with each downstream producer over the input price, taking as given the other input prices r^*_{-i}. Each bargaining pair chooses a price that is a best response to the equilibrium outcomes of other negotiations, where $r^*_1, r^*_2, \ldots, r^*_n$ denotes the Nash-in-Nash bargaining equilibrium. Let α_i denote the bargaining power of input supplier i relative to any downstream producer, where $0 < \alpha_i < 1$ and $i = 1, 2, \ldots, n$.

The timeline of the game is as follows. In stage 1, input suppliers choose supply schedules y_i ($i = 1, 2, \ldots, n$). The competitive entry of producers determines output q. In stage 2, input suppliers and producers bargain bilaterally and select input prices r_i ($i = 1, 2, \ldots, n$). The output market clears at price p.

2.3. *The Bundled-Input Monopoly*

Before examining complementary monopolies selling individual inputs, consider a monopolist that sells a bundle of all of the inputs to the downstream industry. The two-stage game is as follows. In the first stage, the bundled monopolist chooses the size of the bundle of inputs q, and in the second stage, the bundled monopolist negotiates a price ρ for the bundle of inputs with each of the downstream producers. Downstream producers enter the market as long as

[11] Recall that in Cournot's model, each input supplier offers a price to suppliers and then provides whatever amount is demanded by producers.

marginal returns are greater than or equal to the input price: $P(q) - c \geq \rho$. Let α denote the bundled monopolist's bargaining power with downstream producers, where $0 < \alpha \leq 1$.

Consider the second-stage bargaining outcome where q is the size of the input bundle chosen by the monopolist in the first stage. Because of free entry, downstream output equals the size of the input bundle. Bilateral bargaining between the bundled monopolist and each producer solves the asymmetric Nash cooperative bargaining problem:

$$\max_{\rho}[P(q) - c - \rho]^{1-\alpha}\left(\rho - \sum_{i=1}^{n} k_i\right)^{\alpha}.$$

The first-order condition for the price of the bundle is

$$\alpha[P(q) - c - \rho] = (1 - \alpha)\left(\rho - \sum_{i=1}^{n} k_i\right). \tag{5}$$

This implies that for a given downstream output q, the price for the bundle of inputs equals

$$\rho(q) = \alpha[P(q) - c] + (1 - \alpha)\sum_{i=1}^{n} k_i.$$

The monopolist chooses demand for the input bundle, or equivalently downstream output, to maximize profits. Substituting for the price of the bundle of inputs, the monopolist's profit equals

$$\rho(q)q - \sum_{i=1}^{n} k_i q = \alpha\left[P(q) - c - \sum_{i=1}^{n} k_i\right]q. \tag{6}$$

This implies that the bundled-input monopolist chooses downstream output q to maximize

$$\left[P(q) - c - \sum_{i=1}^{n} k_i\right]q \quad \text{for all } \alpha,$$

so the output q^{M} does not depend on bargaining power.

Because the bundled-input monopolist maximizes industry profits, the bundled-monopoly outcome provides an efficiency benchmark that can be used to evaluate the outcome with complementary monopolists. Assume that there exists an interior solution to the monopoly problem, $q^{M} > 0$. The first-order condition for the monopolist's problem is

$$P'(q^{M})q^{M} + P(q^{M}) - c - \sum_{i=1}^{n} k_i = 0. \tag{7}$$

The monopoly profit is positive:

$$\alpha\left\{[P(q^{M}) - c]q^{M} - \sum_{i=1}^{n} k_i q^{M}\right\} = -\alpha P'(q^{M})(q^{M})^2 > 0.$$

The monopolist's output choice need not be unique. If there are multiple solutions, then for ease of notation let q^{M} denote the smallest output. The main result holds whether or not the monopoly output is unique. The monopolist's price for the bundle of inputs equals a weighted average of the marginal return to producers evaluated at the monopoly output and total marginal costs:

$$\rho^{\mathrm{M}} = \alpha[P(q^{\mathrm{M}}) - c] + (1-\alpha)\sum_{i-1}^{n} k_i .$$

By profit maximization, the price for the bundle of inputs is less than or equal to the markup of the final price over producers' unit cost, $\rho^{\mathrm{M}} \leq P(q^{\mathrm{M}}) - c$.

2.4. Equilibrium of the Two-Stage Game

In the first stage, input suppliers choose supply offers represented by $y_i^\star$ ($i = 1, \ldots, n$), and producers' demand for inputs equals $q^\star \leq \min\{y_1^\star, y_2^\star, \ldots, y_n^\star\}$. In the second stage, the Nash-in-Nash equilibrium bargaining outcome is represented by input prices $r_i^\star$ ($i = 1, \ldots, n$), and the final market price is $p^\star = P(q^\star)$. I solve the model by backward induction.

Consider the second-stage bargaining problem given demand for inputs q determined in the first stage. Given the input prices chosen by bargaining between other input suppliers and producers $r_{-i}^\star$, the input price r_i solves the asymmetric Nash bargaining problem for each $i = 1, \ldots, n$:

$$\max_{r_i}\left[P(q) - c - \sum_{j\neq i}^{n} r_j^\star - r_i\right]^{1-\alpha_i}(r_i - k_i)^{\alpha_i} .$$

Letting $r_i = r_i^\star$, the first-order conditions imply that

$$r_i^\star - k_i = \frac{\alpha_i}{1-\alpha_i}\left[P(q) - c - \sum_{j=1}^{n} r_j^\star\right] \qquad (i = 1, \ldots, n). \tag{8}$$

Summing both sides over i gives total prices

$$\sum_{i=1}^{n} r_i^\star = \left[1 \bigg/ \left(1 + \sum_{i=1}^{n}\tfrac{\alpha_i}{1-\alpha_i}\right)\right]\left\{\sum_{i=1}^{n}\tfrac{\alpha_i}{1-\alpha_i}[P(q) - c] + \sum_{i=1}^{n} k_i\right\}. \tag{9}$$

Define β_i:

$$\beta_i = \frac{\alpha_i}{1-\alpha_i}\left[1 \bigg/ \left(1 + \sum_{j=1}^{n}\frac{\alpha_j}{1-\alpha_j}\right)\right] \qquad (i = 1, \ldots, n). \tag{10}$$

Then the first-order conditions give the equilibrium input prices $r_i^\star = r_i^\star(q)$ as functions of industry demand for inputs:

$$r_i^\star(q) = \beta_i\left[P(q) - c - \sum_{j=1}^{n} k_j\right] + k_i \qquad (i = 1, \ldots, n). \tag{11}$$

This establishes that the bargaining equilibrium exists and is unique. Notice that $0 < \beta_i < 1$ and

$$0 < \sum_{i=1} \beta_i < 1$$

for any α_i $(i = 1, 2, \ldots, n)$.

It follows that the equilibrium profit of each input supplier i is

$$V_i(q, r_i^\star) = \beta_i \left\{ [P(q) - c]q - \sum_{j=1}^{n} k_j q \right\} \qquad (i = 1, 2, \ldots, n). \tag{12}$$

At industry demand for inputs q, each producer earns a profit

$$\Pi(q, r_1^\star, r_2^\star, \ldots, r_n^\star) = \left(1 - \sum_{i=1}^{n} \beta_i\right)\left[P(q) - c - \sum_{j=1}^{n} k_i\right]. \tag{13}$$

Consider now the equilibrium of the two-stage game. Proposition 1 presents the main result of the analysis. The result holds whether or not the profit-maximizing monopoly output is unique, where q^{M} is the smallest profit-maximizing monopoly output. Define social welfare as the sum of consumers' and producers' surplus: $W(p) = \mathrm{CS}(p) + \mathrm{PS}(p)$, where consumers' surplus is

$$\mathrm{CS}(p) = \int_{p}^{\infty} D(z)dz$$

and total producers' surplus is

$$\mathrm{PS}(p) = \left(p - c - \sum_{i=1}^{n} k_i\right) D(p).$$

Proposition 1. In the first stage, the weakly dominant strategy equilibrium in supply schedules is unique and equivalent to the profit-maximizing bundled-monopoly output, $y_i^\star = q^{\mathrm{M}}$ $(i = 1, \ldots, n)$, so that equilibrium industry output is $q^\star = q^{\mathrm{M}}$. In the second stage, input prices are unique, $r_i^\star = r_i^\star(q^{\mathrm{M}})$, and the total of input prices is

$$\sum_{i=1}^{n} r_i^\star(q^{\mathrm{M}}) = \left(\sum_{i=1}^{n} \beta_i\right)[P(q^{\mathrm{M}}) - c] + \left(1 - \sum_{i=1}^{n} \beta_i\right) \sum_{j=1}^{n} k_j. \tag{14}$$

The final price equals the joint-profit-maximizing price $P(q^{\mathrm{M}})$, and total input prices are strictly less than the markup over producers' marginal cost:

$$\sum_{i=1}^{n} r_i^\star(q^{\mathrm{M}}) < P(q^{\mathrm{M}}) - c.$$

Consumers' surplus, total producers' surplus, and social welfare in the two-stage game with complementary monopolists are the same as with a bundled monopolist.

This result establishes that with complementary inputs, the noncooperative equilibrium with quantity-setting suppliers yields the cooperative outcome. The proposition shows that the weakly dominant strategy equilibrium is unique even if the bundled-monopoly outcome is not because the equilibrium equals the smallest output that maximizes bundled-monopoly profit. The result depends only on the assumptions that demand is downward sloping and inputs are perfect complements. Notice also that the weakly dominant strategy equilibrium with supply schedules is unique even though there are many Nash equilibria with fixed quantities.

The complementarity of inputs serves as a tacit coordination mechanism. A supplier strictly prefers the bundled-monopoly outcome to any other outcome. This means that a supplier will choose the quantity of an input that would be offered by a bundled monopolist regardless of what other suppliers are offering. If other suppliers offer greater quantities of inputs in comparison to the bundled-monopoly outcome, a supplier strictly prefers to restrict the equilibrium output by offering fewer units of the input. If other suppliers offer smaller quantities of inputs than the bundled-monopoly outcome, a supplier strictly prefers not to restrict the output further and is indifferent between offering the bundled-monopoly quantity and the restricted quantity of inputs.

Because inputs are strict complements, every supplier understands that his offer of an input controls the market outcome under some conditions. So each supplier will choose to offer the quantity of an input that would be offered by a bundled monopolist. In this way, suppliers coordinate without the need for mergers or formal agreements. In addition, notice that bargaining power does not affect the equilibrium output. Regardless of how rents are divided, suppliers have an incentive to choose the optimal output.

Proposition 1 shows that an input supplier has an incentive to choose an upper limit on the quantity supplied. The result also shows that an input supplier would not choose a positive minimum amount because it does not know what other input suppliers are offering. In addition, the result shows that an input supplier would not offer a fixed output rather than a supply schedule because that could result in an offer in excess of the quantity offered by other suppliers and in excess of the amount demanded by downstream producers. Making either a minimum offer or a fixed-output offer would risk costly overproduction.

A bundled-monopoly supplier facing a competitive downstream industry is likely to have all of the market power; that is, $\alpha = 1$. Then the bundled monopolist chooses a per-unit tariff equal to the final markup over the producer's cost, $\bar{\rho}^{\mathrm{M}} = P(q^{\mathrm{M}}) - c$. So when $\alpha = 1$, total input prices with complementary monopolists are strictly less than the bundled-monopoly price:

$$\sum_{i=1}^{n} r_i^* < \bar{\rho}^{\mathrm{M}}.$$

In general, total input prices with bargaining are greater than, equal to, or less than the bundled-monopoly price:

$$\sum_{i=1}^{n} r_i^* > (=)(<) \rho^{\mathrm{M}} \quad \text{as} \quad \sum_{i=1}^{n} \beta_i > (=)(<) \alpha.$$

To compare complementary monopolists with the bundled monopolist, it may be useful to consider the effects of the number of complementary-input monopolists on the outcome of the two-stage game. To examine the effects of more suppliers without changing total costs, suppose that

$$\sum_{i=1}^{n} k_i = K \quad \text{for all } n.$$

It follows that having more suppliers does not affect the equilibrium output q^{M}. Adding more suppliers shifts total bargaining power toward suppliers so that

$$\sum_{i=1}^{n} \beta_i \to 1$$

as n increases, and total prices go to $P(q^{M}) - c$ as n increases. With many suppliers, total per-unit tariffs can exceed the per-unit tariff set by a bundled monopoly with bargaining, assuming that the bargaining power of the bundled monopolist is not affected by the number of inputs and is less than 1.

Suppose that the bargaining power of individual suppliers diminishes with entry, $\alpha_i = 1/n$. Suppose that the bundled-monopoly bargaining power is $\alpha_i > \frac{1}{2}$. Then, as n increases,

$$\sum_{i=1}^{n} \beta_i \to \frac{1}{2},$$

so in the limit total prices are

$$\sum_{i=1}^{n} r_i^{*}(q^{M}) = \frac{1}{2}[P(q^{M}) - c] + \frac{1}{2}\sum_{j=1}^{n} k_j.$$

As the number of complementary monopolists increases, total prices with complementary monopolists will be less than the bundled-monopoly price,

$$\sum_{i=1}^{n} r_i^{*}(q^{M}) < \rho^{M}.$$

The analysis translates into complements in consumption. Suppose that the complementary monopolists sell components used by consumers. A consumer has unit demand for consumption of the set of components $x = \min\{\delta_1, \delta_2, \ldots, \delta_n\}$ with willingness to pay u if x equals one and zero otherwise. Let $G(u)$ denote the cumulative distribution of willingness-to-pay levels across consumers. Suppose that perfectly competitive distributors with operating costs c resell the components to consumers at price p. Then aggregate demand for the set of complements is given by $q = 1 - G(p)$. Aggregate demand is decreasing because the cumulative distribution is necessarily increasing in willingness-to-pay levels. The two-stage game with perfect competition downstream also applies to complements in consumption. Suppliers of complementary products offer supply schedules $Y_i(q)$

in the first stage and bargain over prices r_i with distributors in the second stage. Proposition 1 continues to apply.[12]

2.5. Comparison with the Cournot Outcome

Compare the outcome in the present two-stage game with the outcome of Cournot's posted-price game. In Cournot's model, input suppliers choose prices r_i ($i = 1, \dots, n$), and downstream producers choose how much of the inputs to purchase. The downstream industry is perfectly competitive so that the final output price in the downstream market is

$$p = c + \sum_{i=1}^{n} r_i.$$

Input prices in Cournot's noncooperative equilibrium r_i^{C} ($i = 1, \dots, n$) solve

$$r_i^{\mathrm{C}} = \arg\max_{r_i} (r_i - k_i) D\left(c + \sum_{j \neq i}^{n} r_j^{\mathrm{C}} + r_i\right).$$

In equilibrium, the first-order conditions in Cournot's model are

$$(r_i^{\mathrm{C}} - k_i) D'\left(c + \sum_{j \neq i}^{n} r_j^{\mathrm{C}} + r_i^{\mathrm{C}}\right) + D\left(c + \sum_{j \neq i}^{n} r_j^{\mathrm{C}} + r_i^{\mathrm{C}}\right) = 0. \quad (15)$$

Summing over i implies that

$$\sum_{i=1}^{n} r_i^{\mathrm{C}} - \sum_{i=1}^{n} k_i = -n\left[D\left(c + \sum_{j=1}^{n} r_j^{\mathrm{C}}\right) \Big/ D'\left(c + \sum_{j=1}^{n} r_j^{\mathrm{C}}\right)\right]. \quad (16)$$

The Cournot effect compares total input prices at the noncooperative equilibrium with the bundled-monopoly price. In my setting, this is equivalent to the bundled-monopoly price when the monopolist has all the bargaining power, $\alpha = 1$, which exactly equals the markup over producers' costs,

$$r^{\mathrm{M}} = P(q^{\mathrm{M}}) - c. \quad (17)$$

To see why this is equal to the bundled-monopoly posted price, write the monopolist's first-order condition,

$$P'(q^{\mathrm{M}})q^{\mathrm{M}} + P(q^{\mathrm{M}}) - c - \sum_{i=1}^{n} k_i = 0 \quad \text{as} \quad r^{\mathrm{M}} - \sum_{i=1}^{n} k_i = -P'(q^{\mathrm{M}})q^{\mathrm{M}},$$

and notice that $q^{\mathrm{M}} = D(c + r^{\mathrm{M}})$ and $P'(q^{\mathrm{M}}) = 1/D'(c + r^{\mathrm{M}})$. Then, at the bundled-monopoly price, I have

[12] The setting with complements in consumption also applies to perfect competition when producers have multiunit capacity, which is considered in Section 3, and to monopolistic competition downstream, which is considered in Section 4. The analysis of complements in consumption would change when there is competition from firms supplying substitute products for particular components and when there are imperfect complements so that consumers can purchase subsets of the products.

$$r^{\mathrm{M}} - \sum_{i=1}^{n} k_i = -\frac{D(c + r^{\mathrm{M}})}{D'(c + r^{\mathrm{M}})} < -n\frac{D(c + r^{\mathrm{M}})}{D'(c + r^{\mathrm{M}})}. \qquad (18)$$

Compare the outcome in the posted-price game with the posted-price bundled-monopoly outcome. Suppose that demand is log concave, $d^2 \ln D(p)/dp^2 \leq 0$. This holds, for example, if demand is linear, $D(p) = a/b - p/b$.[13] Given log concavity of demand, comparing the first-order conditions gives the Cournot effect:

$$r^{\mathrm{M}} < \sum_{i=1}^{n} r_i^{\mathrm{C}}.$$

The Cournot effect implies that output is greater with a bundled monopoly than with complementary monopolists:

$$q^{\mathrm{M}} > D\left(c + \sum_{j \neq i}^{n} r_j^{\mathrm{C}}\right) = q^{\mathrm{C}}.$$

This allows a comparison between the two-stage game and Cournot's price-setting game. The following result holds whatever the relative bargaining power of input suppliers and producers in the two-stage bargaining model.

Proposition 2. Let demand be log concave. Then downstream industry output, consumers' surplus, total producers' surplus, and social welfare are greater in the two-stage bargaining game than in Cournot's posted-price game.

Because $q^{\mathrm{M}} > q^{\mathrm{C}}$, the downstream price is lower in the two-stage bargaining game than in Cournot's posted-price game, $P(q^{\mathrm{M}}) < P(q^{\mathrm{C}})$. This implies that $\mathrm{CS}[P(q^{\mathrm{M}})] > \mathrm{CS}[P(q^{\mathrm{C}})]$, and joint-profit maximization implies that $\mathrm{PS}[P(q^{\mathrm{M}})] > \mathrm{PS}[P(q^{\mathrm{C}})]$. So social welfare is greater in the two-stage bargaining game. The result suggests that the Cournot effect is due to the restriction of competitive strategies to posted prices.

There is another interesting difference between the two-stage bargaining game and Cournot's posted-price game. In the two-stage model, when total costs are held constant, the number of complementary-input suppliers does not affect the weakly dominant strategy equilibrium output. So in the two-stage game when total costs are held constant, entry of additional input suppliers does not affect equilibrium output or social welfare. In Cournot's posted-price game, an increase in the number of complementary-input suppliers increases the sum of input prices when demand is log concave. This is because having a greater number of suppliers worsens the free-rider effects of noncooperative competition. This means that in Cournot's model, having a greater number of input suppliers reduces both equilibrium output and social welfare.

[13] Discussions of the standard Cournot quantity model assume that inverse demand $P(q)$ is log concave; see, for example, Amir and Lambson (2000). These results apply to complementary monopolies when demand $D(p)$ is log concave.

3. Complementary Monopolies with Perfect Competition and Multiunit Capacity in the Downstream Market

This section considers complementary monopolies with perfect competition in the downstream market when downstream firms have multiunit capacity and increasing costs. In the first stage, input suppliers offer aggregate supply schedules y_i and choose the number of contracts m_i. Producers enter the market and choose input demands x. In the second stage, input supplier and producer pairs engage in Nash-in-Nash bargaining over transfers t_i.

3.1. Complementary Monopolies When Producers Have Multiunit Capacity

Each of the m downstream producers supplies x units of a homogeneous product. Excluding the costs of inputs supplied by the complementary monopolists, each downstream producer has a cost function $C(x)$ that is twice differentiable, increasing, and convex. To characterize competitive entry, assume that average cost $\mathrm{AC}(x) = C(x)/x$ is U-shaped with minimum efficient scale at output x_0. As before, market inverse demand is $p = P(q)$, and market demand is $q = D(p)$, where q is industry output.

Inputs are perfect complements, and producers enter sequentially so that they are able to obtain the same amount of each of the inputs.[14] Define $y_{\min}$ equal to $\min\{y_1, y_2, \ldots, y_n\}$ and $m_{\min}$ equal to $\min\{m_1, m_2, \ldots, m_n\}$. Because inputs are perfect complements, market output is limited by the aggregate supply of inputs $q \leq y_{\min}$, and entry is limited by the number of supply contracts $m \leq m_{\min}$.

Producers' transfers to input suppliers are $t_1, t_2, \ldots, t_n$. Given industry output q, each downstream producer has the profit function

$$\Pi(q, x, t_1, t_2, \ldots, t_n) = \max_x \left[P(q)x - C(x) - \sum_{i=1}^{n} t_i \right]. \tag{19}$$

Each producer's input demand is limited by the aggregate supply of inputs divided by the number of suppliers, $x \leq y_{\min}/m$. So input demand is $x = \min\{X[P(q)], y_{\min}/m\}$, where the unconstrained input demand $X = X[P(q)]$ solves

$$P(q) = C'(X). \tag{20}$$

Producers have an incentive to enter the market as long as $\Pi(q, x, t_1, t_2, \ldots, t_n) \geq 0$.

Each input supplier i earns profits

$$V_i(q, m, t_i) = mt_i - qk_i \qquad (i = 1, 2, \ldots, n). \tag{21}$$

Input suppliers are active if and only if $V_i(q, m, t_i) \geq 0$.

Define social welfare as the sum of consumers' and producers' surplus:

$$W(p, x) = \mathrm{CS}(p) + \mathrm{PS}(p, x), \tag{22}$$

[14] Each producer has a Leontief production function $x = \min\{\zeta_1, \zeta_2, \ldots, \zeta_n\}$, where ζ_i is the amount of input i used by the producer.

where consumers' surplus is

$$\mathrm{CS}(p) = \int_p^\infty D(z)dz$$

and total producers' surplus is

$$\mathrm{PS}(p,\ x) = pD(p) - mC(x) - \sum_n^n k_i D(p).$$

The timeline of the game is as follows. First, input suppliers choose input supply schedules y_i ($i = 1, 2, \ldots, n$). They also choose the number of contracts m_i ($i = 1, 2, \ldots, n$). The competitive entry of producers determines the number of producers m, and producers choose input demands x. Second, input suppliers and producers bargain bilaterally and select transfers t_i ($i = 1, 2, \ldots, n$). The output market clears at price p.

The equilibrium of the two-stage game is as follows. In the first stage, equilibrium suppliers' aggregate input offers are y_i^* ($i = 1, \ldots, n$), and contract offers are m_i^* ($i = 1, \ldots, n$). Competitive entry of producers determines the number of producers $m^* \leq \min\{m_1^*, m_2^*, \ldots, m_n^*\}$, producers' profit maximization determines input demands x^*, and aggregate input demand is $q^* = m^* x^* \leq \min\{y_1^*, y_2^*, \ldots, y_n^*\}$. In the second stage, Nash-in-Nash bargaining between input supplier and producer pairs determines transfers t_i^* ($i = 1, \ldots, n$), and the market clears at price $p^* = P(q^*)$.

3.2. *The Bundled-Input Monopoly*

The bundled-monopoly problem is useful for characterizing the two-stage bargaining game with multiunit capacity. In the first stage, the bundled monopolist chooses the size of the input bundle q^{M} and the number of contracts m^{M}. The bundled monopolist and each producer bargain over a transfer T^{M}. The bundled monopolist's total profit depends on industry output, the number of producers, and the transfer:

$$V^{\mathrm{M}}(q,\ m,\ T) = mT - \sum_{j=1}^n k_j q. \tag{23}$$

Each downstream producer has the profit function $\Pi(q, T) = P(q)x - C(x) - T$.

I solve the bundled monopolist's problem by backward induction. In the second stage, the bundled monopolist bargains with each producer, taking as given the input bundle q^{M}, the number of contracts m^{M}, and each producer's demand for the bundle of inputs $x^{\mathrm{M}} = \min\{X[P(q^{\mathrm{M}})],\ q^{\mathrm{M}}/m^{\mathrm{M}}\}$. The asymmetric Nash cooperative-bargaining problems are

$$\max_T [P(q^{\mathrm{M}})x^{\mathrm{M}} - C(x^{\mathrm{M}}) - T]^{1-\alpha}\left(T - \sum_{j=1}^n k_j x^{\mathrm{M}}\right)^{\alpha}.$$

The first-order conditions for each bargaining problem imply that

$$T = \alpha[P(q^{\mathrm{M}})x^{\mathrm{M}} - C(x^{\mathrm{M}})] + (1-\alpha)\sum_{j=1}^n k_j x^{\mathrm{M}}. \tag{24}$$

Substituting for T, I find that the bundled monopolist's profit is

$$V^{M}(q^{M}, m^{M}, T) = \alpha\left[P(q^{M})m^{M}x^{M} - m^{M}C(x^{M}) - \sum_{j=1}^{n} k_j m^{M}x^{M}\right]. \qquad (25)$$

Consider now the first-stage equilibrium. Substituting for the transfer payment in the producer's profit, I obtain

$$\Pi(q, T) = (1-\alpha)\left[P(q)x - C(x) - \sum_{j=1}^{n} k_j x\right]. \qquad (26)$$

Because the bundled monopolist maximizes profit, it follows that the producer's profit is nonnegative. This implies that entry continues until the number of producers equals the number of contracts, $m = m^{M}$.

In addition, because the producer's profit is nonnegative, the price is greater than average cost, so the price is greater than average cost at the minimum efficient scale: $P(q) > \mathrm{AC}(x) \geq \mathrm{AC}(x_0)$. Because marginal cost is increasing, the unconstrained input demand is increasing in the output price, $X'[P(q)] = 1/C''(X) > 0$. This implies that the unconstrained input demand is greater than the producer's minimum efficient scale, $X[P(q)] > x_0$.

The bundled monopolist chooses the size of the input bundle q^{M} and the number of contracts m^{M} to maximize profit $V^{M}(q, m, T)$. Substituting for the transfer, I find that the bundled monopolist's profit is

$$V^{M}(q, m, T) = \alpha\left[P(q)q - mC(x) - \sum_{i=1}^{n} k_i q\right], \qquad (27)$$

where $x = \min\{X[P(q)], q/m\}$. For any input bundle q, the bundled monopolist chooses the number of contracts m to minimize costs. So each producer's input demand is constrained, $x = q/m$, and the profit-maximizing number of contracts m^{M} solves

$$C\left(\frac{q}{m}\right) - C'\left(\frac{q}{m}\right)\frac{q}{m} = 0. \qquad (28)$$

This implies that producers operate at minimum efficient scale, $x = q/m = x_0$.

The bundled monopolist chooses output to maximize

$$\alpha\left[P(q)q - q\mathrm{AC}(x_0) - \sum_{i=1}^{n} k_i q\right].$$

The monopoly output satisfies the first-order condition:

$$P(q^{M}) + P'(q^{M})q^{M} - \mathrm{AC}(x_0) - \sum_{i=1}^{n} k_i = 0. \qquad (29)$$

Assume that there exists a profit-maximizing output, and for ease of notation let q^{M} denote the smallest profit-maximizing monopoly output. The market equilibrium number of producers is $m^{M} = q^{M}/x_0$.

The bundled monopolist receives

$$T^{M} = \alpha[P(q^{M})x_0 - C(x_0)] + (1-\alpha)\sum_{j=1}^{n} k_j x_0.$$

The transfer can be expressed as a two-part tariff, $T^{M} = \Gamma^{M} + \rho^{M}x$, consisting of a per-unit tariff equal to total marginal cost

$$\rho^{M} = \sum_{j=1}^{n} k_j$$

and a lump-sum tariff equal to a share of the producer's profit

$$\Gamma^{M} = \alpha\left[P(q)x_0 - C(x_0) - \sum_{j=1}^{n} k_j x_0\right].$$

3.3. Equilibrium of the Two-Stage Game

Consider now the two-stage game with complementary-input monopolists when producers have multiunit capacity. Bilateral bargaining between each input monopolist and each producer solves the asymmetric Nash-in-Nash cooperative-bargaining problem:

$$\max_{t_i}\left[P(q)x - C(x) - \sum_{j\neq i}^{n} t_j^{*} - t_i\right]^{1-\alpha_i}[t_i - k_i x]^{\alpha_i},$$

where $x = \min\{X[P(q)], q/m\}$.

Equilibrium transfers $t_i^{*} = t_i^{*}(q, x)$ are then

$$t_i^{*}(q, x) = \beta_i\left[P(q)x - C(x) - \sum_{j=1}^{n} k_j x\right] + k_i x \qquad (i = 1, \ldots, n). \tag{30}$$

This establishes that the bargaining equilibrium exists and is unique given q and x.

Proposition 3 shows that the two-stage bargaining game with two-part tariffs achieves the cooperative outcome. As before, the result holds whether or not the bundled-monopoly profit-maximizing output is unique.

Proposition 3. In the first stage, the weakly dominant strategy equilibrium in supply schedules is unique and equivalent to the profit-maximizing bundled-monopoly output $y_i^{*} = q^{M}$ $(i = 1, \ldots, n)$. The weakly dominant strategy equilibrium in supply contracts is unique and equivalent to the profit-maximizing bundled monopoly in supply contracts: $m_i^{*} = m^{M}$ $(i = 1, \ldots, n)$. Producers operate at minimum efficient scale so that in equilibrium, industry input demand is $q^{*} = q^{M}$, output per producer is $x^{M} = x_0$, and the number of producers is $m^{M} = q^{M}/x_0$. In the second stage, transfers are unique and equal:

$$t_i^{*} = \beta_i\left[P(q^{M})x_0 - C(x_0) - \sum_{j=1}^{n} k_j x_0\right] + k_i x_0. \tag{31}$$

The final price equals the joint-profit-maximizing price $P(q^M)$, and total transfers are

$$\sum_{i=1}^{n} t_i^* = \sum_{i=1}^{n} \beta_i [P(q^M)x_0 - C(x_0)] + \left(1 - \sum_{i=1}^{n} \beta_i\right) \sum_{j=1}^{n} k_j x_0. \quad (32)$$

Total transfers are strictly less than each producer's revenue net of production costs at the bundled-monopoly output:

$$\sum_{i=1}^{n} t_i^* < P(q^M)x_0 - C(x_0).$$

Consumers' surplus, total producers' surplus, and social welfare in the two-stage bargaining game with supply schedules and supply contracts are the same as with a bundled monopolist.

The intuition for this result is as follows. Each input supplier obtains a share of aggregate profits and has an incentive to choose input supply offers and input supply contracts that maximize aggregate profits. Because inputs are perfect complements, for any aggregate input offer $y_{\min}$, the game in supply contracts m_i generates the minimum efficient scale for producers. The profit-maximizing number of contracts is unique because the minimum efficient scale is unique with a U-shaped average cost curve.

The number of contracts m that gives the minimum efficient scale for each supplier—that is, $m = y_{\min}/x_0$—is a weakly dominant strategy for each supplier. This is because the equilibrium number of contracts is less than or equal to the smallest number of contracts offered by suppliers. If all other suppliers choose a greater number of contracts such that input demand is lower than minimum efficient scale, a supplier can lower the number of contracts so that input demand attains minimum efficient scale. Conversely, if all other suppliers choose contracts that limit input demand below minimum efficient scale, a supplier would not choose to lower the number of contracts further and would be indifferent between a greater number of contracts and the number of contracts associated with minimum efficient scale. So for any aggregate input supply offer, suppliers choose the number of contracts such that producers' input demands are at minimum efficient scale.

In turn, this implies that input suppliers choose aggregate input supply offers to maximize profits, anticipating that the number of input contracts will be such that producers operate at minimum efficient scale. As in the preceding setting with unit capacity, this implies that input supply offers maximize joint industry profits. This follows from the perfect complementarity of inputs as before. The joint-profit-maximizing input supply offers are the unique weakly dominant strategy.

Nash-in-Nash bargaining in the second stage implies that all active producers earn nonnegative profits, $\Pi(q, x, t_1, t_2, \ldots, t_n) \geq 0$. So entry of downstream producers continues until $m = m_{\min}$. The final price is greater than average cost at the minimum efficient scale, $P(q^M) > AC(x_0)$ so that input demands are constrained and each producer operates at minimum efficient scale.

With bargaining, total transfers are greater than, equal to, or less than the bundled-monopoly transfer:

$$\sum_{i=1}^{n} t_i^* > (=)(<) T^M \quad \text{as} \quad \sum_{i=1}^{n} \beta_i > (=)(<) \alpha.$$

As noted previously, a bundled-monopoly supplier facing a competitive downstream industry is likely to have all of the market power; that is, $\alpha = 1$. Then the bundled monopolist chooses a transfer equal to producers' returns net of total marginal costs, $\overline{T}^M = P(q^M)x_0 - C(x_0)$. This implies that total lump-sum tariffs with complementary monopolists are strictly less than the bundled-monopoly lump-sum tariff

$$\sum_{i=1}^{n} t_i^* < \overline{T}^M.$$

3.4. Comparison with the Cournot Outcome

This section extends the Cournot complementary-monopolies pricing game to allow input suppliers to offer two-part tariffs to producers. The equilibrium with two-part tariffs is given by r_i^C, R_i^C $(i = 1, \ldots, m)$. Each input supplier chooses a two-part tariff to maximize profits, $(r_i - k_i)q + mR_i$. Let ρ_0^M, Γ_0^M denote the profit-maximizing two-part tariff for the bundled input monopoly. Producers take as given the two-part tariffs offered by input suppliers.

Each producer has a profit function

$$\Pi(q, r_1, r_2, \ldots, r_n, R_1, R_2, \ldots, R_n) = px - C(x) - \sum_{i=1}^{n} r_i x - \sum_{i=1}^{n} R_i. \quad (33)$$

Each producer's input demand

$$x = X\left(p - \sum_{i=1}^{n} r_i\right)$$

solves the first-order condition

$$p = C'(x) + \sum_{i=1}^{n} r.$$

Entry occurs until each active producer has a profit of 0:

$$P(mx)x = C(x) + \sum_{i=1}^{n} r_i x + \sum_{i=1}^{n} R_i. \quad (34)$$

Combine the entry condition with producers' first-order conditions. Then per-unit tariffs cancel from marginal and average costs, so that each producer operates at the minimum average cost output:

$$C'(x^A) = \frac{1}{x^A}\left[C(x^A) + \sum_{i=1}^{n} R_i\right]. \quad (35)$$

The producers' minimum average cost output depends on total lump-sum tariffs but not on per-unit tariffs,

$$x^{A} = x^{A}\left(\sum_{i=1}^{n} R_i\right),$$

and is increasing in total lump-sum tariffs,

$$x^{A'}\left(\sum_{i=1}^{n} R_i\right) = \frac{1}{C''(x^{A})x^{A}} > 0.$$

If lump-sum tariffs are 0, it follows that each producer operates at the minimum average cost output $x^{A}(0) = x_0$.

The minimum average cost price is

$$p^{A} = \mathrm{AC}(x^{A}) + \sum_{i=1}^{n} r_i + \frac{1}{x^{A}}\sum_{i=1}^{n} R_i.$$

This implies that industry output is a function of total lump-sum tariffs, $q = D(p^{A})$. The number of producers is equal to industry output divided by output per producer, $m = q/x^{A}$, which is a function of total lump-sum tariffs and total per-unit tariffs. Define the Cournot output with the option of two-part tariffs:

$$q^{C} = D\left[\mathrm{AC}(x^{A}) + \sum_{j=1}^{n} r_j^{C} + \frac{1}{x^{A}}\sum_{i=1}^{n} R_i^{C}\right].$$

I compare the Cournot posted-price game with the bundled-input monopolist when input suppliers have the option of offering two-part tariffs. The proposition shows that the Cournot effect continues to hold.

Proposition 4.

a) The bundled-input monopolist sets the lump-sum tariff equal to 0, $R^{M} = 0$, and the per-unit tariff for the bundle of inputs r^{M} is greater than total marginal costs and solves

$$D[\mathrm{AC}(x_0) + r^{M}] + \left(r^{M} - \sum_{i=1}^{n} k_i\right)D'[\mathrm{AC}(x_0) + r^{M}] = 0. \qquad (36)$$

Downstream output is $q_0^{M} = D[\mathrm{AC}(x_0) + r^{M}]$.

b) In the equilibrium of the posted-price game, each complementary-input supplier sets the lump-sum tariff equal to 0, $R_i^{C} = 0$ $(i = 1, \dots, n)$, and per-unit tariffs r_i^{C} are greater than marginal cost and solve the modified Cournot first-order conditions

$$D\left[\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_j^{C}\right] + (r_i^{C} - k_i)D'\left[\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_j^{C}\right] = 0 \qquad (i = 1, \dots, n). \quad (37)$$

Downstream output is

$$q^{C} = D\left[\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_j^{C}\right].$$

c) Let demand be log concave. The Cournot effect holds that

$$r^{\mathrm{M}} < \sum_{i=1}^{n} r_i^{\mathrm{C}}.$$

Downstream output, consumers' surplus, total producers' surplus, and social welfare are greater with a bundled monopolist than with two or more complementary-input suppliers.

Total per-unit tariffs are increasing in the number of input suppliers. So final output, consumers' surplus, total producers' surplus, and social welfare are decreasing in the number of input suppliers.

Compare the first-order conditions for the bundled monopolist in the two-stage bargaining game and the posted-price game. With posted two-part tariffs, the bundled monopolist chooses a lump-sum tariff of 0 and a per-unit tariff $r^{\mathrm{M}} = P(q_0^{\mathrm{M}}) - C(x_0)/x_0$ that satisfy the first-order condition

$$P(q_0^{\mathrm{M}}) + P'(q_0^{\mathrm{M}})q_0^{\mathrm{M}} - \mathrm{AC}(x_0) - \sum_{i=1}^{n} k_i = 0. \qquad (38)$$

This is the same as for the bundled monopolist in the two-stage bargaining game, so output levels are the same: $q^{\mathrm{M}} = q_0^{\mathrm{M}}$.

Producers operate at minimum efficient scale in the two-stage bargaining game with a bundled monopolist or complementary monopolists. Producers also operate at minimum efficient scale with posted two-part tariffs with either a bundled monopolist or complementary monopolists. So with a bundled monopolist, the number of producers is the same with two-stage bargaining and posted prices: $m^{\mathrm{M}} = m_0^{\mathrm{M}} = q_0^{\mathrm{M}}/x_0$.

However, with complementary monopolists, total output is greater with two-stage bargaining than with posted prices, $q^* = q_0^{\mathrm{M}} > q^{\mathrm{C}}$, when the Cournot effect holds. This implies that when there are complementary monopolists, the number of producers is greater with two-stage bargaining than with posted prices: $m^* = q^*/x_0 > m^{\mathrm{C}} = q^{\mathrm{C}}/x_0$.

The bundled monopolist with posted two-part tariffs earns profit greater than or equal to that of the bundled monopolist in the two-stage bargaining game:

$$P(q_0^{\mathrm{M}})q_0^{\mathrm{M}} - m_0^{\mathrm{M}}C(x_0) - \sum_{i=1}^{n} k_i q_0^{\mathrm{M}} \geq V^{\mathrm{M}}(q^{\mathrm{M}}, m^{\mathrm{M}}, t^{\mathrm{M}}).$$

The profit levels are equal when the bundled monopolist has all of the bargaining power in the two-stage game, $\alpha = 1$.

Social welfare with a bundled monopolist is the same in the two-stage bargaining game and in the posted-price game. It follows that social welfare with complementary monopolists in the two-stage game is greater than in the posted-price game:

$$W[P(q^{\mathrm{M}}),\ x_0] - W[P(q^{\mathrm{C}}),\ x_0] = \int_{q^{\mathrm{C}}}^{q^{\mathrm{M}}} \left[P(q) - \mathrm{AC}(x_0) - \sum_{i=1}^{n} k_i \right] dq > 0. \qquad (39)$$

This implies the following result.

Proposition 5. The final output, the number of producers, and input demand with a bundled monopolist are the same in the two-stage bargaining game and in the posted-price game with the option of two-part tariffs. When demand is log concave, final output, consumers' surplus, total producers' surplus, and social welfare with complementary monopolies are greater in the two-stage bargaining game than in the posted-price game with the option of two-part tariffs.

In Cournot's posted-price game, a larger number of complementary monopolists increases free-rider effects. The number of complementary monopolists does not affect the outcome of the two-stage game when total marginal costs are constant. So as the number of complementary monopolists increases, the difference between welfare in the two-stage bargaining game and the posted-price game increases as well.

4. Complementary Monopolies with Oligopoly Competition in the Downstream Market

This section considers complementary monopolies that sell to oligopoly producers. In the first stage, each input supplier i chooses a supply schedule $Y_i(q)$ represented by y_i^* $(i = 1, \ldots, n)$, and total producer demand for inputs is $q^* = \min\{y_1^*, y_2^*, \ldots, y_n^*\}$. In the second stage, a Nash-in-Nash bargaining game determines two-part tariffs r_i^*, R_i^* $(i = 1, \ldots, n)$.

4.1. Producers

There are m downstream producers each offering a differentiated product x_j $(j = 1, 2, \ldots, m)$. Each of the downstream producers sells multiple units of output. Each producer has a Leontief production function $x_j = \min\{\zeta_1, \zeta_2, \ldots, \zeta_n\}$, where ζ_i is the amount of input i. Let $q = y_{\min}$ be the minimum of the maximum input supply offers and assume that all active producers obtain the same amount of the inputs. Then each input supplier faces the constraint $x_j \leq q/m$.

Each producer j chooses the price p_j and has demand $x_j = D(p_j, p_{-j}; m)$ $(j = 1, 2, \ldots, m)$. Assume that when prices are symmetric, demand per producer $x(p; m) = D(p_j, p_{-j}; m)$ is strictly decreasing in the market price p and that $P(x; m)$ denotes the inverse of demand per producer $x(p; m)$. The slope of each producer's demand with symmetric prices is $z(p; m) = \partial D(p_j, p_{-j}; m)/\partial p_j < 0$. Assume that products are substitutes so that the market price effect on each producer's demand is greater than the own-price effect on demand, $z(p, m) < x_p(p, m)$.[15]

Producers engage in Bertrand-Nash price competition with differentiated products. Producers have unit costs c excluding the costs of purchased inputs. Assume that market equilibrium prices are symmetric and the producers' price strategy

[15] Demand per producer is decreasing in the market price, $x_p(p, m) < 0$ (Vives 2000, 2008).

$$p^* = p^*\left(\sum_{i=1}^{n} r_i + c;\ m\right)$$

is increasing in per-unit costs

$$\sum_{i=1}^{n} r_i + c.$$

These properties can be derived from standard assumptions about market demand (see Vives 2008; Spulber 2013). The reduced-form model of oligopoly competition among producers follows Vives (2005, 2008).

When producers do not face input constraints, each producer's first-order condition for the symmetric equilibrium price p^* can be written

$$\left(p^* - c - \sum_{i=1}^{n} r_i\right) z(p^*;\ m) + x(p^*;\ m) = 0. \tag{40}$$

The equilibrium net returns for each producer are then

$$\Pi(q, r_1, r_2, \ldots, r_n, R_1, R_2, \ldots, R_n) = \left(p^* - c - \sum_{i=1}^{n} r_i\right) x(p^*;\ m) - \sum_{i=1}^{n} R_i, \tag{41}$$

where

$$p^* = p^*\left(\sum_{i=1}^{n} r_i + c;\ m\right).$$

Each producer demands a quantity

$$x\left[p^*\left(\sum_{i=1}^{n} r_i + c;\ m\right);\ m\right]$$

of each input.

If each producer faces a binding input constraint, that is,

$$x\left[p^*\left(\sum_{i=1}^{n} r_i + c;\ m\right);\ m\right] \geq \frac{q}{m},$$

each producer's demand for inputs is q/m. The market equilibrium price solves $x(p; m) = q/m$ so that $p = P(q/m; m)$. So with capacity constraints, we can write the equilibrium net returns for each producer as

$$\Pi(q, r_1, r_2, \ldots, r_n, R_1, R_2, \ldots, R_n) = \left[p\left(\frac{q}{m};\ m\right) - c - \sum_{i=1}^{n} r_i\right]\frac{q}{m} - \sum_{i=1}^{n} R_i. \tag{42}$$

4.2. The Bundled-Input Monopoly

To characterize the complementary-monopolies problem, it is useful to consider a monopolist that sells the bundle of inputs to the downstream industry. The bundled monopolist uses a per-unit tariff $\rho \geq 0$ and a lump-sum tariff $\Gamma \geq$

0. Letting total transfers per producer be $t = \rho(q/m) + \Gamma$, bilateral bargaining solves the asymmetric Nash cooperative bargaining problem

$$\max_t \left\{ \left[P\left(\frac{q}{m};\, m\right) - c \right] \frac{q}{m} - t \right\}^{1-\alpha} \left(t - \sum_{i=1}^{n} k_i \frac{q}{m} \right)^{\alpha}.$$

The first-order condition for the bargaining problem is then

$$\alpha \left\{ \left[p\left(\frac{q}{m};\, m\right) - c \right] \frac{q}{m} - t \right\} = (1-\alpha)\left(t - \sum_{i=1}^{n} k_i \frac{q}{m} \right). \tag{43}$$

This gives the total transfer per producer as a function of downstream output:

$$t^{\mathrm{M}}(q) = \alpha \left[P\left(\frac{q}{m};\, m\right) - c \right] \frac{q}{m} + (1-\alpha) \sum_{i=1}^{n} k_i \frac{q}{m}. \tag{44}$$

The bundled monopolist's profit then equals a share of downstream industry profits:

$$mt^{\mathrm{M}}(q) - \sum_{i=1}^{n} k_i q = \alpha \left[P\left(\frac{q}{m};\, m\right) - c - \sum_{i=1}^{n} k_i \right] q. \tag{45}$$

The monopolist's problem is expressed in terms of total input demand q, where the per-unit input tariff ρ solves $x[p^*(\rho + c;\, m);\, m] = q/m$, and the output price is $p = P(q/m;\, m)$.

The solution to the bundled monopolist's profit-maximization problem does not depend on bargaining power. Assuming the existence of an interior solution q^{M}, I find that the first-order condition for the monopolist's problem is

$$P\left(\frac{q^{\mathrm{M}}}{m};\, m\right) + P'\left(\frac{q^{\mathrm{M}}}{m};\, m\right) \frac{q^{\mathrm{M}}}{m} - c - \sum_{i=1}^{n} k_i = 0. \tag{46}$$

As before, the solution need not be unique. Let $q^{\mathrm{M}} > 0$ be the smallest profit-maximizing input-demand level, again for ease of notation. The equilibrium output price is $p^{\mathrm{M}} = P(q^{\mathrm{M}}/m;\, m)$.

The bundled monopolist's per-unit tariff ρ^{M} is determined by the equilibrium output per firm $x[p^*(\rho^{\mathrm{M}} + c;\, m);\, m] = q^{\mathrm{M}}/m$. From the producers' first-order conditions, the per-unit tariff for the bundle of inputs is equal to the final market price minus unit cost plus the ratio of the demand per producer to the slope of each producer's demand:

$$\rho^{\mathrm{M}} = p^* - c + \frac{x(p^*;\, m)}{z(p^*;\, m)}. \tag{47}$$

Because the bundled monopolist cannot capture all returns using marginal cost pricing and lump-sum tariffs, there is some double marginalization. From the bundled monopolist's first-order condition and $q = mx(p^*;\, m)$, the per-unit tariff is

$$\rho^{\mathrm{M}} = \sum_{i=1}^{n} k_i - P'\left(\frac{q^{\mathrm{M}}}{m};\, m\right) \frac{q^{\mathrm{M}}}{m} + \frac{1}{z(p^*;\, m)} \frac{q^{\mathrm{M}}}{m}. \tag{48}$$

Because products are substitutes, it follows that $-P'(q/m;\, m) = -1/x_p(p^*,\, m) > -1/z(p^*,\, m)$. This implies that the bundled monopolist's per-unit tariff is greater than total marginal cost,

$$\rho^{\mathrm{M}} > \sum_{i=1}^{n} k_i,$$

which implies double marginalization.

The monopolist's lump-sum tariff for the bundle of inputs equals the total transfer net of revenue from per-unit tariffs if that is positive: $\Gamma^{\mathrm{M}} = \max\{0,\, t^{\mathrm{M}}(q^{\mathrm{M}}) - \rho^{\mathrm{M}}(q^{\mathrm{M}}/m)\}$. If $t^{\mathrm{M}}(q^{\mathrm{M}}) > \rho^{\mathrm{M}}(q^{\mathrm{M}}/m)$, substituting for the per-unit tariff gives

$$\Gamma^{\mathrm{M}} = -(1-\alpha)\left[P\left(\frac{q}{m};\, m\right) - c - \sum_{i=1}^{n} k_i\right]\frac{q}{m} - \left(\frac{x(p^*;\, m)}{z(p^*;\, m)}\right)\frac{q}{m}. \tag{49}$$

Substituting from the bundled monopolist's first-order condition implies that the lump-sum tariff is

$$\Gamma^{\mathrm{M}} = \left[(1-\alpha)P'\left(\frac{q^{\mathrm{M}}}{m};\, m\right) - \frac{1}{z(p^*;\, m)}\right]\left(\frac{q^{\mathrm{M}}}{m}\right)^2 \tag{50}$$

for $1/(\alpha - 1) < P'(q^{\mathrm{M}}/m;\, m)z(p^*;\, m)$, and $\Gamma^{\mathrm{M}} = 0$ otherwise.

4.3. Equilibrium of the Two-Stage Game

At the first stage, input suppliers choose supply schedules $Y_1(q), Y_2(q), \ldots, Y_n(q)$ to maximize net benefits

$$V_i(q,\, r_1,\, r_2,\, \ldots,\, r_n,\, R_1,\, R_2,\, \ldots,\, R_n) = r_i q + mR_i - C(q), \tag{51}$$

where $q = y_{\min}$. Input suppliers will participate only if they receive nonnegative net benefits $V_i(q, r_1, r_2, \ldots, r_n, R_1, R_2, \ldots, R_n) \geq 0$.

At the second stage, each input supplier bargains bilaterally with each producer. The Nash-in-Nash equilibrium of the bargaining stage is represented by $r_1^*, r_2^*, \ldots, r_n^*, R_1^*, R_2^*, \ldots, R_n^*$. Denote the total transfer from a producer to an input supplier $t_i = r_i(q/m) + R_i$. Then input suppliers have net benefits

$$V_i(q,\, r_1,\, r_2,\, \ldots,\, r_n,\, R_1,\, R_2,\, \ldots,\, R_n) = mt_i - k_i q \qquad (i = 1, \ldots, n). \tag{52}$$

The equilibrium net returns for each producer are

$$\Pi(q,\, r_1,\, r_2,\, \ldots,\, r_n,\, R_1,\, R_2,\, \ldots,\, R_n) = \left[P\left(\frac{q}{m};\, m\right) - c\right]\frac{q}{m} - \sum_{i=1}^{n} t_i. \tag{53}$$

Given the transfers chosen by bargaining between other input suppliers with producers t_{-i}^*, each transfer t_i solves the Nash bargaining problem

$$\max_{t_i}\left\{\left[P\left(\frac{q}{m};\, m\right) - c\right]\frac{q}{m} - \sum_{j\neq i}^{n} t_j^* - t_i\right\}^{1-\alpha_i}\left(t_i - k_i\frac{q}{m}\right)^{\alpha_i} \qquad (i = 1, \ldots, n).$$

I now characterize the equilibrium of the two-stage bargaining game when there is oligopoly competition in the downstream market.

Proposition 6. In the first stage, the weakly dominant strategy equilibrium in supply schedules is unique and equivalent to the smallest profit-maximizing bundled-monopoly output: $y_i^* = q^M$ $(i = 1, \ldots, n)$. In the second stage, transfers are unique, $t_i^* = t_i^*(q^M)$, and the total of transfers per producer is

$$\sum_{i=1}^{n} t_i^*(q^M) = \sum_{i=1}^{n} \beta^i \left[P\left(\frac{q^M}{m}; m\right) - c\right]\frac{q^M}{m} + \left(1 - \sum_{i=1}^{n} \beta_i\right)\sum_{j=1}^{n} k_j \frac{q^M}{m}.$$

The final output price equals that with a bundled monopoly $P(q^M/m; m)$, and total transfers are less than the producers' markups over producers' costs times output per producer:

$$\sum_{i=1}^{n} t_i^*(q^M) < \left[P\left(\frac{q^M}{m}; m\right) - c\right]\frac{q^M}{m}.$$

Consumers' surplus, total producers' surplus, and social welfare in the two-stage bargaining game with supply schedules are the same as with a bundled monopolist.

With oligopoly competition downstream, complementary monopolists achieve the bundled-monopoly output, which is the cooperative outcome. Mergers are not necessary for complementary monopolists to achieve the cooperative output. Depending on the relative bargaining power, total transfers are less than, equal to, or greater than bundled-monopoly revenues:

$$m\sum_{i=1}^{n} t_i^*(q^M) < (=)(>) \rho^M q^M + m\Gamma^M \quad \text{as} \quad \sum_{i=1}^{n} \beta_i < (=)(>) \alpha.$$

If the bundled monopoly has sufficient bargaining power relative to producers in comparison with complementary monopolists, then total transfers will be greater with the bundled monopolist than with complementary monopolists.

5. Antitrust Policy Implications

This section considers some antitrust policy implications of the two-stage bargaining model with complementary monopolists. First, I consider antitrust policy toward conglomerate mergers among complementary monopolists. Second, I consider antitrust policy toward conglomerate mergers when the downstream producer is a monopsony. Third, I examine the problem of successive monopolies and antitrust policy toward vertical mergers. Finally, I show how two-stage bargaining applies to basic bilateral monopoly.

5.1. Conglomerate Mergers: Implications of Bargaining versus Posted Prices

The present results have implications for antitrust policy toward conglomerate mergers of complementary monopolists. Complementary-input monopolists can achieve the cooperative outcome by offering supply schedules to producers and bargaining over prices. The resulting output will equal the bundled-monopoly output, and total input prices will be strictly less than the producers' markup over costs at the monopoly output. The analysis shows that the presence of complementarities in production or in consumption need not justify conglomerate mergers. This means that a conglomerate merger of complementary monopolists need not generate any benefits that would result from bundling.

In contrast, it has been argued on the basis of the Cournot effect that a merger of firms offering complementary goods would increase social welfare. The merged firms could reduce prices by bundling complementary goods, which would eliminate the effects of posted-price competition that existed before the merger. According to the Organisation for Economic Co-operation and Development (OECD 2001), the Cournot effect would justify a merger of firms offering complementary goods if premerger prices were above competitive levels and the merged firm would have a significant market share or would engage in tying or bundling of the complementary goods.[16]

The Cournot effect relies on particular assumptions about the conduct of complementary monopolists. It depends on the assumptions that complementary monopolists use posted prices when selling inputs to producers and that suppliers choose prices noncooperatively. As a consequence, input suppliers do not take into account the effects of their prices on the profits of other complementary monopolists, which leads to total input prices above the bundled-monopoly level. The preceding discussion shows that the Cournot effect holds when complementary monopolists offer prices to producers with unit capacity or when complementary monopolists can offer two-part tariffs to producers with multiunit capacity.

However, the Cournot effect does not hold with bargaining between input suppliers and producers. The dependence of the hypothetical Cournot effect on specific competitive conduct limits its use as a justification for mergers. The effect cannot be a defense of conglomerate mergers unless it can also be established that before the merger, companies indeed engage in competition with posted prices. A conglomerate merger thus need not generate benefits from product bundling.

The absence of a Cournot effect does not in itself rule out such mergers. In practice, conglomerate mergers may offer various cost economies associated with consolidation of production or transactions. However, conglomerate mergers may also create problems resulting from reduced competition, as discussed in the

[16] According to the Organisation for Economic Co-operation and Development (OECD 2012, p. 8), "In addition to efficiency effects there is a less obvious reason why a merger uniting complements could lead to lower prices. Such a merger could also internalise the effects of lowering the price of one complement on sales and profits earned on another. This Cournot effect will not exist or be significant unless pre-merger prices were above competitive levels in at least one of the complements. Another necessary condition is that the merged entity will either have a significant market share in at least one of the complements in which there were pre-merger supracompetitive pricing, or will engage in some form of tying, bundling or analogous practice having the effect of internalising a pricing externality in complementary products."

US Department of Justice's (DOJ's) nonhorizontal merger guidelines.[17] The policy implication of the present analysis is that antitrust policy makers should focus on how the merger would affect costs, prices, and competitive behavior without necessarily relying on the presence of complementarities.

The Cournot effect played a significant role in antitrust policy toward the proposed merger of GE and Honeywell (see Choi 2001; Vives and Staffiero 2009). Both GE and Honeywell supplied complementary inputs such as engines and avionics to aircraft producers. There are many reasons to suppose that GE and Honeywell did not rely on noncooperative posted prices as a means of selling components to aircraft producers. It is more likely that they engaged in bilateral contract negotiations with aircraft producers to specify supply schedules, demand orders, prices, and other contract terms. The companies would be more likely to rely on bargaining because of the small number of companies involved, the high cost of inputs, and the need to establish production and delivery schedules. In addition, they would rely on contracts because of the investments needed to manufacture engines and other components and to produce final outputs. In addition, contracts would be necessary to address the complex technological issues associated with product quality, interoperability of components, and allocation of intellectual property.

Although the DOJ approved the proposed $43 billion merger, the European Commission (EC) rejected it. The EC decision directly addresses the Cournot effect (Commission Decision 2004/134, 2001 O.J. [L 48] 91–92). The companies seeking to merge argued that aircraft engines and components such as avionics were complements and that the merger would facilitate bundling, which would lower final prices. In its decision, the EC states (2001 O.J. [L 48] 92), "Therefore, even if the demand for aircraft at the industry level were inelastic, i.e., even in the face of a price reduction by all entities for the product bundle, it did not increase sufficiently to render price reduction profitable[;] the Commission's investigation has indicated that a price reduction of the bundled system by the merged entity is likely to shift customers' demand away from competitors to the merged entity's bundled product." The EC expresses concerns that the merger would increase the market power of the merging companies in jet engines for commercial, regional, and corporate jets and for components such as avionics.

The EC considered the Cournot effect without performing sufficient theoretical or empirical analysis to determine whether that effect was applicable to the market in question. The European Court of First Instance reviewed the EC decision and various presentations by economists, noting that "the question as to whether the Cournot effect would have given the merged entity an incentive to engage in mixed bundling in the present case is a matter of controversy" (Case T-210/01, *General Electric Co. v. Comm'n*, 2005 E.C.R. II-05575, II-5740).

According to the Court of First Instance (2005 E.C.R. II-5733), the EC argued, "[I]t follows from well-established economic theories, particularly the 'Cournot

[17] See US Department of Justice, Non-horizontal Merger Guidelines (https://www.justice.gov/atr/non-horizontal-merger-guidelines).

effect' . . . , that the merged entity would have an economic incentive to engage in the practices foreseen by the Commission and that there was no need to rely on a specific economic model in that regard." The Court of First Instance (2005 E.C.R. II-5742) found that "by merely describing the economic conditions which would in its view exist on the market after the merger, the Commission did not succeed in demonstrating, with a sufficient degree of probability, that the merged entity would have engaged in mixed bundling after the merger."

Manufacturers of aircraft engines and components and assemblers of aircraft would be likely to specify input supply and demand commitments and to bargain over input prices. So the present analysis suggests that even with strict complements and complementary monopolies, the Cournot effect need not be observed. This suggests that evaluating the competitive effects of the GE-Honeywell merger would require additional economic analysis.

The effect of conglomerate mergers when there are complements in consumption is affected by the structure of consumers' preferences and the presence of competitors. Choi (2008) extends the analysis of the Cournot effect to include mixed bundling, which involves the merged firm selling complementary components both separately and as a bundle. Choi (2008) finds that mergers can have positive or negative effects on social welfare depending on consumers' preferences and how the merger affects competitors.

Free-rider effects can arise through complementary activities other than the supply of complementary inputs. For example, competition in research-and-development (R&D) investments, product features, or advertising could generate inefficiencies. Mergers potentially would affect inefficiencies resulting from these other types of noncooperative behavior. There are other market institutions that could address potential free-rider effects from complementary activities. For example, companies cooperate in R&D through joint ventures and cross licensing of intellectual property. Companies can coordinate technology standard setting through standard-setting organizations. Companies also cooperate to coordinate some types of complementary advertising through trade associations.

5.2. Conglomerate Mergers When the Downstream Producer Is a Monopolist

How should antitrust policy view mergers of suppliers when there is a monopsony buyer? If all suppliers of inputs were to merge and sell to a monopsony buyer, the result would be a bilateral monopoly. The main antitrust issue is whether downstream market power should be a defense for upstream mergers.

This question has been studied when suppliers provide inputs that are substitutes. Blair and Harrison (1993) and Hovenkamp (1991) raise various objections to the countervailing power defense for mergers. Campbell (2007) addresses these arguments and finds that mergers to monopoly create social benefits when the buyer is a monopsony. Because the merged input suppliers would bargain with the downstream monopsony, the resulting supply of inputs would maximize joint returns.

In this section, I consider whether a conglomerate merger of complementary-input suppliers would create similar social benefits when there is a monopsony buyer. The conglomerate merger would create a bundled-monopoly seller. The result would again be a bilateral monopoly in the bundle of complementary inputs. The bundled-monopoly seller and the monopsony buyer would be able to bargain and choose a supply of inputs that would maximize joint returns.

The question is whether such a conglomerate merger would be necessary. Suppose first that input suppliers can offer two-part tariffs to the downstream monopolist. An equilibrium of the posted-price game would be for input suppliers to set per-unit tariffs at marginal cost and then to choose lump-sum tariffs. In equilibrium, the total lump-sum tariffs would equal the monopoly producer's profit, and the joint-profit maximum would be achieved. The monopolist would choose output to maximize profits,

$$\Pi^{\mathrm{M}} = \max_{q}\left(P(q)q - cq - \sum_{i=1}^{n} k_i q\right),$$

and total lump-sum tariffs would sum to monopoly profits,

$$\sum_{i=1}^{n} R_i = \Pi^{\mathrm{M}}.$$

The problem with this scenario is that there are infinitely many outcomes in which total lump-sum tariffs equal monopoly profits. It is not evident that the upstream complementary monopolists would be able to achieve the coordination necessary to implement an equilibrium with two-part tariffs.

The posted-price game would seem to suggest the need for a conglomerate merger as a means of coordinating the upstream input suppliers. The merged firm would charge a per-unit tariff equal to total marginal cost, and the merged firm and the downstream monopsony would bargain over the lump-sum tariff.

However, the present setting suggests that Nash-in-Nash bargaining between each of the upstream complementary monopolists and the downstream monopsony would be sufficient to achieve a joint optimum. Bargaining would result in a unique division of surplus. Total lump-sum tariffs would be

$$\sum_{i=1}^{n} R_i(q^{\mathrm{M}}) = \left(\sum_{i=1}^{n} \beta_i\right)[P(q^{\mathrm{M}}) - c]q^{\mathrm{M}} + \left(1 - \sum_{i=1}^{n} \beta_i\right)\sum_{j=1}^{n} k_j q^{\mathrm{M}}. \tag{54}$$

This suggests that bilateral bargaining would eliminate the potential benefits of a conglomerate merger of upstream complementary monopolists. There is no need for a conglomerate merger of input suppliers to generate a bilateral monopoly with the downstream producer.

5.3. *Successive Monopolies and Vertical Mergers*

It has been argued that vertical integration by multiple successive monopolists avoids the problem of double or multiple marginalization. There has been extensive discussion of the problem of successive monopoly in the economics literature.[18] *United States v. Aluminum Co. of America* (*Alcoa*) (148 F.2d 416, 437 [2d Cir. 1945]) is a classic example of an antitrust case alleging a successive monopoly because the company produced both aluminum ingots and aluminum sheets.[19]

When products are complements in demand, companies have an incentive to bundle the products. This often raises antitrust policy concerns about tying. However, McChesney (2015) argues that many cases with complementary products should not be treated as tying because they are more accurately described as successive monopolies. McChesney points out that this applies to the cases of *United States v. Microsoft Corp*. (253 F.3d 34 [DC Cir. 2001]), *Jefferson Parish Hospital District No. 2 v. Hyde* (466 US 2 [1984]), and *Concord v. Edison Electric Co.* (915 F.2d 17 [1st Cir. 1990]). In *Microsoft,* the complementary products were the Windows operating system and the Internet Explorer browser. In *Jefferson Parish,* the complementary products were hospital medical services and anesthesia. In *Concord,* Boston Edison both produced and distributed electric power.

Just as the Cournot effect has been applied to justify conglomerate mergers, so the successive-monopoly model has been applied to justify vertical mergers.[20] Vertical integration avoids the problem of double or multiple marginalization because the vertically integrated firm efficiently prices internally produced inputs at their marginal costs. For example, Spengler (1950, p. 352) argues that "vertical integration, if unaccompanied by a competition-suppressing amount of horizontal integration and if conducive to cost and price reduction, should be looked upon with favor by a court interested in lower prices and a better allocation of resources." Alternatively, it is argued that breaking up a vertically integrated company would cause welfare losses by leading to double marginalization if there is a monopoly at two or more vertical levels.

When successive monopolies involve just two firms, joint-profit maximization can be achieved with two-part pricing. The upstream firm can offer the downstream firm a per-unit tariff equal to marginal cost and a lump-sum tariff equal to the downstream firm's surplus. However, the problem becomes more complicated with multiple vertical stages.

The present two-stage bargaining model suggests that successive monopolies need not lead to welfare losses from double or multiple marginalization. The

[18] For example, Machlup and Taber (1960, p. 107) note that "Wicksell's exposition is enlivened by a picturesque illustration, drawn from a reference by Babbage . . . to the only existing possessor of the skill of making dolls' eyes who sells to the only manufacturer of dolls." Machlup and Taber are quoting Wicksell (1927, p. 276), who is quoting Babbage (1832, pp. 199–20). Babbage's example of successive monopoly in doll making predates Cournot's book.

[19] Alcoa was said to have a monopoly in virgin aluminum ingots although there were foreign suppliers of ingots and recycled aluminum. Alcoa also faced competition from other producers of aluminum sheets. This led to charges of a price squeeze of competitors in aluminum sheets to whom Alcoa supplied ingots.

[20] Timing issues have complicated the economic analysis of successive monopolies. With simultaneous pricing, the outcome is the same as Cournot's complementary-monopolies model, so the final price exceeds the joint-profit-maximizing price because of multiple marginalization. With sequential pricing, the outcome is the standard double-marginalization result, which again departs from the joint-profit maximum. The final prices can differ as a consequence of timing differences, but in each situation the final price exceeds the joint-monopoly price.

upstream and downstream monopolists can coordinate through noncooperative supply schedules and bargaining over prices. This suggests that a merger of successive monopolies is not necessary for reducing final prices. Conversely, a breakup of a vertically integrated firm need not increase final prices.

To illustrate the basic issues, suppose that there are three successive monopolies, with costs k_i ($i = 1, 2, 3$). The same analysis applies with two, three, or more vertical levels. Each successive monopoly supplies a necessary input that is used in fixed proportions to produce the input at the next level of production. The output of competitive producers in the final market is q, and each successive monopoly offers an input supply schedule $Y_i(q)$ given by $Y_i(q) = \min\{q, y_i\}$ ($i = 1, 2, 3$).

Suppose that the final market is perfectly competitive and final producers have unit costs c. The primary successive monopolist makes no transfer payment and has profit $V_1 = t_2 - k_1 q$. The secondary successive monopolist makes a transfer payment of t_2 to the primary successive monopolist and has profit $V_2 = t_3 - k_2 q - t_2$. The final successive monopolist makes a transfer payment t_3 to the second successive monopolist and sells to competitive producers in the final market, obtaining a profit $V_3 = P(q)q - cq - k_3 q - t_3$.

The Nash-in-Nash bargaining equilibrium consists of the transfers t_2^* and t_3^*. The primary monopolist bargains with the secondary monopolist over the transfer t_2 and with relative bargaining power α_2:

$$\max_{t_2} (t_3^* - k_2 q - t_2)^{1-\alpha_2} (t_2 - k_1 q)^{\alpha_2}.$$

The secondary monopolist bargains with the final monopolist over the transfer t_3 with relative bargaining power α_3:

$$\max_{t_3} [P(q)q - cq - k_3 q - t_3]^{1-\alpha_3} (t_3 - k_2 q - t_2^*)^{\alpha_3}.$$

Given the final market output q, the equilibrium transfers are

$$t_2^* = k_2 q + \frac{\alpha_2 \alpha_3}{1 - \alpha_2 + \alpha_2 \alpha_3} \left[P(q)q - cq - \sum_{i=1}^{3} k_i q \right] \tag{55}$$

and

$$t_3^* = k_1 q + k_2 q + \frac{\alpha_3}{1 - \alpha_2 + \alpha_2 \alpha_3} \left[P(q)q - cq - \sum_{i=1}^{3} k_i q \right]. \tag{56}$$

This implies that the successive monopolists have the following profits:

$$V_1 = \frac{\alpha_2 \alpha_3}{1 - \alpha_2 + \alpha_2 \alpha_3} \left[P(q)q - cq - \sum_{i=1}^{3} k_i q \right], \tag{57}$$

$$V_2 = \frac{\alpha_3 - \alpha_2 \alpha_3}{1 - \alpha_2 + \alpha_2 \alpha_3} \left[P(q)q - cq - \sum_{i=1}^{3} k_i q \right], \tag{58}$$

and

$$V_3 = \frac{1-\alpha_2-\alpha_3+\alpha_2\alpha_3}{1-\alpha_2+\alpha_2\alpha_3}\left[P(q)q - cq - \sum_{i=1}^{3} k_i q\right]. \quad (59)$$

Vertical mergers do not improve the market outcome. The bargaining game generates the same output that would be observed if the three successive monopolists were to merge vertically. By reasoning similar to that in proposition 1, I obtain the following result.

Proposition 7. In the first stage, the weakly dominant strategy equilibrium in supply schedules is unique and equivalent to the profit-maximizing bundled-monopoly output $y_i^* = q^M$ $(i = 1, 2, 3)$, so that equilibrium industry input demand is $q^* = q^M$. In the second stage, transfers are unique, $t_i^* = t_i^*(q^M)$ $(i = 1, 2)$, and total transfers are

$$t_2^*(q^M) + t_3^*(q^M) = (\alpha_3 + \alpha_2\alpha_3)[P(q) - c]q + (1 - \alpha_2 - \alpha_3)(k_1 + k_2 + k_3)q.$$

The final price equals the joint-profit-maximizing price $P(q^M)$, and total transfers are strictly less than the markup over producers' marginal cost:

$$t_2^*(q^M) + t_3^*(q^M) < P(q^M) - c.$$

Consumers' surplus, total producers' surplus, and social welfare in the two-stage bargaining game with supply schedules are the same as with a vertically integrated monopolist.

Successive monopolies with fixed proportions are thus identical to the complementary-monopolies model, where the number of levels corresponds to the number of complementary inputs. Just as the manufacturer purchases the input, the input supplier can be viewed as purchasing manufacturing services. The input suppliers and the manufacturer divide the rents from selling to the downstream market because the successive monopolists cannot transact with the competitive downstream market without transacting with each other. The result implies that eliminating successive monopolies need not be a justification for vertical mergers.

5.4. Bilateral Monopoly

Antitrust scholars have long debated whether bilateral monopoly is efficient (see, for example, Friedman 1986; Blair and DePasquale 2015).[21] Economists have come to recognize that bilateral monopoly maximizes joint profits when the two parties bargain over quantities and prices. Bargaining is efficient for any outcome along the contract curve as shown by Edgeworth (1881) and Pareto (1903, 1971).[22] This result also is consistent with axiomatic game theory, which suggests

[21] Horn and Wolinsky (1988) find that incentives for mergers are affected by bargaining in a duopoly model in which each producer has a dedicated supplier.

[22] Tarascio (1972) considers the origins of the Edgeworth-Bowley box and identifies Pareto's critical initial contribution. Coase (1960) observes that bargaining over externalities should generate efficient outcomes when there are no transaction costs and small numbers of agents.

that cooperative behavior should lead to maximization of joint benefits.[23] Noncooperative bargaining also generates Pareto-optimal outcomes; see Rubinstein (1982), which implies that noncooperative bargaining between bilateral monopolists would maximize joint profits.

One way to attain the cooperative outcome is for the upstream monopolist to make a take-it-or-leave-it offer of a two-part tariff. The two-part tariff attains the cooperative outcome because the per-unit tariff equals the upstream firm's marginal cost, and the lump-sum tariff captures the downstream firm's surplus. This would generate an outcome at one end of the contract curve.

Bilateral monopoly is equivalent to the complementary-monopolies problem with $n = 2$, because one seller can be viewed as selling part of the downstream market to the other seller. Machlup and Taber (1960, p. 103) point out that Marshall (1907) notes this equivalence.[24] Many economists have discussed bilateral monopoly and complementary monopoly, so the issues raised in these discussions are closely related.[25]

The present analysis suggests an alternative two-stage bargaining mechanism for bilateral decentralized market exchange. In bargaining between a buyer and a seller, the quantity purchased is strictly complementary to the quantity sold. If a buyer and a seller propose supply schedules to each other, noncooperative bargaining generates efficient outcomes as a unique weakly dominant strategy equilibrium.

Suppose that the monopsonistic buyer has a willingness to pay for output q given by $[P(q) - c]q$. The monopolistic seller can provide output q at a cost of kq. Let q^M be the smallest output that maximizes joint profit $[P(q) - c - k]q$. In the first stage, the buyer and seller each make maximum offers of the amount to be exchanged equal to y_1 and y_2, respectively. The quantity of output to be exchanged is given by the minimum of the two values, $q = \min\{y_1, y_2\}$.

In the second stage, the buyer and seller bargain over the transfer payment. Let α be the buyer's bargaining power, $0 < \alpha < 1$. The buyer's profit is $\alpha[P(q) - c - k]q$, and the seller's profit is $(1 - \alpha)[P(q) - c - k]q$. Proposition 1 implies that the weakly dominant strategy equilibrium is unique, and output is given by

[23] On unrestricted bargaining in game theory, see Shapley (1952), Aumann (1987), Aumann and Shapley (1974), and Shubik (1982, 1984). The axiomatic approach includes, for example, Nash's (1950, 1953) bargaining framework, although Rubinstein (1982, p. 98) points out that "[i]t was Nash himself who felt the need to complement the axiomatic approach . . . with a non-cooperative game."

[24] "Marshall, for example, mentions Cournot's illustration of the monopolists supplying the copper and zinc needed to make brass, and adds his own illustration of spinners and weavers supplying complementary services in the production of cloth, without examining whether or not the more obviously vertical arrangement in his case makes any essential difference" (Machlup and Taber 1960, p. 103). The connection between complementary monopolies and bilateral monopoly was also noted in Zeuthen (1930).

[25] Economists who have analyzed the closely related problems of bilateral monopoly and successive monopoly include Edgeworth (1881), Pareto (1903, 1971), Pigou (1908), Schumpeter (1927), Henderson (1940), Leontief (1946), Fellner (1947), and Morgan (1949). See Hayek's (1934) discussion of Menger ([1871] 1934) on isolated exchange ; see also Wicksell ([1934] 2007). Böhm-Bawerk (1891) studies supply and demand in terms of buyer and seller pairs.

$y_1^* = y_2^* = q^M$. The equilibrium input price per unit is $r^* = (1 - \alpha)[P(q^M) - c] + \alpha k$.

This basic setting can be interpreted as the standard problem of bilateral exchange in which a buyer and seller propose maximum amounts that they wish to purchase or sell. Suppose that there is a numeraire commodity and the buyer has an endowment ω of the numeraire. The buyer's benefit is $B(q) + \omega - rq$, and the seller's benefit is $(r - k)q$, where q is the good produced by the seller. Let q^* be the socially optimal output, $B'(q^*) = C'(q^*)$. Proposition 1 implies that the weakly dominant strategy equilibrium is unique, and output is given by $y_1^* = y_2^* = q^*$. The equilibrium price is $r = (1 - \alpha)[B(q^*)/q^*) - c] + \alpha k$.

6. Conclusion

Strategic interaction involving a combination of noncooperative supply offers and bargaining over prices can generate an efficient outcome. With two-stage bargaining as in the present model, complementary monopolists will maximize joint profits, and the final market price will not exceed the monopoly levels. Efficiency with bargaining over supply schedules holds whether the downstream market is perfectly competitive or monopolistically competitive.

Models that arbitrarily limit noncooperative interaction to posted prices or posted two-part tariffs remove degrees of freedom. With posted-price competition as in Cournot, input prices and final output prices will exceed the monopoly level. The present discussion suggests that the Cournot effect is due to restrictive assumptions about competitive strategies rather than complementarities or input monopolies.

With different market institutions such as negotiation of supply contracts, competition among complementary monopolists can be consistent with joint-profit maximization. Predictions based on the Cournot effect need not hold when complementary monopolists engage in general competitive interactions with supply schedules and price negotiation. Antitrust policy makers should not assume that vertical and conglomerate mergers will increase economic efficiency by eliminating multiple marginalization. In addition, conglomerate mergers of suppliers leading to bilateral monopoly are not necessary for economic efficiency. Economic performance with complementary monopolists depends on market institutions and the nature of strategic interactions.

Appendix

Proofs

A1. Proof of Proposition 1

Supplier i's profit in the first stage of the game is

$$v_i(y_i, y_{-i}) = \beta_i \left\{ [P(q) - c]q - \sum_{j=1}^{n} k_j q \right\},$$

where $q = y_{\min}$ and $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$. For any y_{-i}, let $\bar{y} = \min\{y_{-i}\}$. The profit-maximizing industry output may or may not be unique. Suppose first that the profit-maximizing monopoly output q^{M} is unique. Consider first the possibility that $\bar{y} \geq q^{\mathrm{M}}$. Then, because the monopolist selling the bundle of inputs maximizes profit, it follows that $v_i(q^{\mathrm{M}}, y_{-i}) \geq v_i(y_i, y_{-i})$ for all y_i. If $y_i = q$,

$$v_i(y_i, y_{-i}) = \beta_i \left\{ [P(q) - c]q - \sum_{j=1}^{n} k_j q \right\}.$$

So if $\bar{y} \geq q^{\mathrm{M}}$, supplier i maximizes profit by choosing the monopoly output $y_i^{*} = q^{\mathrm{M}}$. Conversely, if $\bar{y} < q^{\mathrm{M}}$, then because the bundled monopolist maximizes profit, it follows that $v_i(q^{\mathrm{M}}, y_{-i}) \geq v_i(y_i, y_{-i})$ for all y_i and strictly for $y_i < \bar{y}$. Again, supplier i maximizes profit by choosing the bundled-monopoly output $y_i^{*} = q^{\mathrm{M}}$. This implies that the bundled-monopoly output is the unique weakly dominant strategy for each supplier i, and thus the unique weakly dominant strategy equilibrium is the bundled-monopoly output.

Now suppose that the profit-maximizing bundled-monopoly output is not unique, and let q' and q'' be monopoly outputs, where $q' < q''$. If $q' < \bar{y} < q''$, then supplier i strictly prefers to offer the lower monopoly output to any other offer, $y_i^{*} = q'$. If $q'' \leq \bar{y}$, then supplier i is indifferent between the two monopoly outputs. If $\bar{y} \leq q'$, then the supplier is indifferent between q' and $\bar{y}$ and strictly prefers q' to any $y_i < \bar{y}$. Therefore, the smallest profit-maximizing bundled-monopoly output q^{M} is the weakly dominant strategy for each supplier i. Summing input prices evaluated at q^{M} gives

$$\sum_{i=1}^{n} r_i^{*} = \left(\sum_{i=1}^{n} \beta_i \right) [P(q^{\mathrm{M}}) - c] + \left(1 - \sum_{i=1}^{n} \beta_i \right) \sum_{j=1}^{n} k_j .$$

Total per-unit tariffs satisfy

$$\sum_{i=1}^{n} r_i^{*} < P(q^{\mathrm{M}}) - c$$

because bundled-monopoly profit is positive;

$$[P(q^{\mathrm{M}}) - c]q^{\mathrm{M}} > \sum_{j=1}^{n} k_j q^{\mathrm{M}} \quad \text{and} \quad \sum_{i=1}^{n} \beta_i < 1.$$

Q.E.D.

A2. Proof of Proposition 3

The game is solved by backward induction. The bargaining stage gives total transfers

$$t_i^{*}(q, x) = \beta_i \left[P(q)x - C(x) - \sum_{j=1}^{n} k_j x \right] + k_i x.$$

So complementary monopolists have profit equal to $V_i = m t_i^{*} - k_i q$ so that

$$V_i = \beta_i \left[P(q)q - q\text{AC}(x) - \sum_{j=1}^{n} k_j q \right].$$

This means that $V_i/\beta_i = V^{\text{M}}(q, m, \rho, \Gamma)/\alpha$.

Because $V_i \geq 0$, competitive entry continues until $m = \min\{m_1, m_2, \ldots, m_n\}$. In addition, because the producer's profit is nonnegative, $P(q) > \text{AC}(x) \geq \text{AC}(x_0)$. Increasing marginal cost implies that the unconstrained input demand is greater than at minimum efficient scale, $X[P(q)] > x_0$. The demand for inputs is $x = \min\{X[P(q)], q/m\}$.

For any y_{-i}, let $\bar{y} = \min\{y_{-i}\}$, where $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$. For any m_{-i}, let $\bar{m} = \min\{m_{-i}\}$, where $m_{-i} = (m_1, \ldots, m_{i-1}, m_{i+1}, \ldots, m_n)$. Supplier i's profit in the first stage of the game is

$$V_i(y_i, y_{-i}, m_i, m_{-i}) = \beta_i \left[P(y_{\min})y_{\min} - y_{\min}\text{AC}\left(\frac{y_{\min}}{m_{\min}}\right) - \sum_{j=1}^{n} k_j q \right].$$

Given $\{y_1, y_2, \ldots, y_n\}$, define $m_0 = y_{\min}/x_0$. Consider first the possibility that $\bar{m} \geq m_0$. Then, because average cost has a minimum at x_0, $V_i(y_i, y_{-i}, m_0, m_{-i}) \geq V_i(y_i, y_{-i}, m_i, m_{-i})$ for all m_i. Conversely, if $\bar{m} < m_0$, then because average cost has a minimum at x_0, it follows that $V_i(y_i, y_{-i}, m_0, m_{-i}) \geq V_i(y_i, y_{-i}, m_i, m_{-i})$ for all m_i and strictly for $m_i < \bar{m}$. This implies that m_0 is the unique weakly dominant strategy for each supplier i and thus is the unique weakly dominant strategy equilibrium. So, given free entry, the number of suppliers is $m_0 = y_{\min}/x_0$, so that $x = x_0$ for all $y_{\min}$. Suppliers operate at minimum efficient scale, and input demands are constrained.

Now I can write supplier i's profit as

$$v_i(y_i, y_{-i}) = \beta_i \left[P(y_{\min})y_{\min} - y_{\min}\text{AC}(x_0) - \sum_{j=1}^{n} k_j y_{\min} \right].$$

Applying reasoning similar to that in the proof of proposition 1, I find that this implies that in the first stage, the weakly dominant strategy equilibrium in supply schedules is unique, $y_i^* = q^{\text{M}}$ $(i = 1, \ldots, n)$, and $q^* = q^{\text{M}}$. This uniquely determines output per producer x^{M} and transfers t_i^* $(i = 1, \ldots, n)$. Because producers earn positive net returns, entry implies that $m^{\text{M}} = q^{\text{M}}/x_0$. Because

$$\sum_{i=1}^{n} \beta_i < 1,$$

I have

$$\sum_{i=1}^{n} t_i^* < P(q^{\text{M}})x_0 - C(x_0).$$

Social welfare is $W(p, x_0) = \text{CS}(p) + \text{PS}(p, x_0)$, where consumers' surplus is

$$\text{CS}(p) = \int_p^\infty D(z)dz,$$

and, because $mC(x_0) = \mathrm{AC}(x_0)D(p)$, total producers' surplus is

$$\mathrm{PS}(p,\ x_0) = \left[p - \mathrm{AC}(x_0) - \sum_{i=1}^{n} k_i\right] D(p).$$

Q.E.D.

A3. Proof of Proposition 4

Note that $q = mx^{\mathrm{A}}$ and

$$q^{\mathrm{C}} = D\left[\mathrm{AC}(x^{\mathrm{A}}) + \sum_{j=1}^{n} r_j^{\mathrm{C}} + \frac{1}{x^{\mathrm{A}}}\sum_{i=1}^{n} R_i^{\mathrm{C}}\right].$$

I can write the complementary monopolists' profit-maximization problems as follows:

$$\max_{r_i,\,R_i}\left(r_i - k_i + \frac{R_i}{x^{\mathrm{A}}}\right) D\left[\frac{C(x^{\mathrm{A}})}{x^{\mathrm{A}}} + \sum_{j\neq i}^{n} r_j^{\mathrm{C}} + r_i + \left(\sum_{j\neq i}^{n} R_j^{\mathrm{C}} + R_i\right) \Big/ x^{\mathrm{A}}\right],$$

where

$$x^{\mathrm{A}} = x^{\mathrm{A}}\left(\sum_{j\neq i}^{n} R_j^{\mathrm{C}} + R_i\right) \qquad (i = 1, \ldots, n).$$

The first-order conditions are

$$D + \left(r_i - k_i + \frac{R_i}{x^{\mathrm{A}}}\right) D' = 0$$

and

$$\left[\frac{x^{\mathrm{A}} - R_i x^{\mathrm{A}\prime}}{(x^{\mathrm{A}})^2}\right] D + \left(r_i - k_i + \frac{R_i}{x^{\mathrm{A}}}\right) D' \frac{\partial p^{\mathrm{A}}}{\partial R_i} = 0.$$

Combining the first-order conditions and noting that

$$p^{\mathrm{A}} = \mathrm{AC}(x^{\mathrm{A}}) + \sum_{j=1}^{n} r_j^{\mathrm{C}} + \frac{1}{x^{\mathrm{A}}}\sum_{i=1}^{n} R_i^{\mathrm{C}}$$

gives

$$\frac{x^{\mathrm{A}} - R_i x^{\mathrm{A}\prime}}{(x^{\mathrm{A}})^2} = \frac{\partial p^{\mathrm{A}}}{\partial R_i} = \frac{1}{(x^{\mathrm{A}})^2}\left\{[C'(x^{\mathrm{A}})x^{\mathrm{A}\prime} + 1]x^{\mathrm{A}} - \left[C(x^{\mathrm{A}}) + \sum_{i=1}^{n} R_i\right] x^{\mathrm{A}\prime}\right\}.$$

This implies that

$$R_i^{\mathrm{C}} = -C'(x^{\mathrm{A}})x^{\mathrm{A}} + C(x^{\mathrm{A}}) + \sum_{j=1}^{n} R_j,$$

so $R_i^{\mathrm{C}} = 0$ for all i. So each producer operates at minimum efficient scale, $x^{\mathrm{A}} = x_0$.

Because lump-sum tariffs equal 0, complementary monopolists choose r_i to maximize

$$(r_i - k_i)D\left[\mathrm{AC}(x_0) + \sum_{j \neq i}^{n} r_j^{\mathrm{C}} + r_i\right].$$

Per-unit input tariffs r_i^{C} solve the modified Cournot first-order conditions

$$D\left[\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_j^{\mathrm{C}}\right] + (r_i^{\mathrm{C}} - k_i)D'\left(\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_j^{\mathrm{C}}\right) = 0.$$

Summing over n gives the following condition:

$$\sum_{i=1}^{n} r_i^{\mathrm{C}} - \sum_{i=1}^{n} k_i = -n\left(\left\{D\left[\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_i^{\mathrm{C}}\right]\right\} \Big/ \left\{D'\left[\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_i^{\mathrm{C}}\right]\right\}\right).$$

I can derive the bundled-input monopolist's profit-maximizing choices in the same way. It follows that the bundled-input monopolist chooses a lump-sum tariff of 0, $R^{\mathrm{M}} = 0$, and the price of the bundle r^{M} solves

$$\max_r \left(r - \sum_{i=1}^{n} k_i\right) D[\mathrm{AC}(x_0) + r].$$

The price of the bundle satisfies the first-order condition

$$r^{\mathrm{M}} - \sum_{i=1}^{n} k_i = -\frac{D[\mathrm{AC}(x_0) + r^{\mathrm{M}}]}{D'[\mathrm{AC}(x_0) + r^{\mathrm{M}}]}.$$

By log concavity of demand,

$$\sum_{i=1}^{n} r_i^{\mathrm{C}} > r^{\mathrm{M}}.$$

So

$$q_0^{\mathrm{M}} = D[\mathrm{AC}(x_0) + \rho_0^{\mathrm{M}}] > q^{\mathrm{C}} = D\left[\mathrm{AC}(x_0) + \sum_{j=1}^{n} r_j^{\mathrm{C}}\right].$$

This implies that $\mathrm{CS}[P(q_0^{\mathrm{M}})] > \mathrm{CS}[P(q^{\mathrm{C}})]$, $\mathrm{PS}[P(q_0^{\mathrm{M}}), x_0] > \mathrm{PS}[P(q^{\mathrm{C}}), x_0]$, and $W[P(q_0^{\mathrm{M}}), x_0] > W[P(q^{\mathrm{C}}), x_0]$. Q.E.D.

A4. Proof of Proposition 6

The first-order conditions for the Nash cooperative bargaining solution imply that

$$t_j^* - k_i \frac{q}{m} = \frac{\alpha_i}{1 - \alpha_i}\left[P\left(\frac{q}{m}; m\right)\frac{q}{m} - c\frac{q}{m} - \sum_{j=1}^{n} t_j^*\right] \qquad (i = 1, \ldots, n).$$

Summing both sides over i gives

$$\sum_{j=1}^{n} t_j^* = 1 \Big/ \left[1 + \sum_{j=1}^{n} (\alpha_i / 1 - \alpha_i)\right] \times \left\{\sum_{j=1}^{n} \frac{\alpha_i}{1 - \alpha_i}\left[P\left(\frac{q}{m}; m\right) - c\right]\frac{q}{m} + \sum_{j=1}^{n} k_j \frac{q}{m}\right\}.$$

The equilibrium transfers that result from Nash bargaining are

$$t_j^* = \beta_i \left[P\left(\frac{q}{m};\, m\right) - c - \sum_{j=1}^{n} k_j \right] \frac{q}{m} + k_i \frac{q}{m} \qquad (i = 1, \ldots, n),$$

where the weights β_i are the same as before. The transfers $t_i^* = t_i^*(q)$ are unique functions of industry demand for inputs.

It follows that at industry demand q for inputs, the equilibrium profit of each input supplier i is

$$V_i(q,\, t_1^*,\, t_2^*,\, \ldots,\, t_n^*) = \beta_i \left[P\left(\frac{q}{m};\, m\right) q - cq - \sum_{j=1}^{n} k_j q \right] \qquad (i = 1,\, 2,\, \ldots,\, n).$$

Supplier i's profit in the first stage of the game is

$$v_i\left(y_i,\, y_{-i}\right) = \beta_i \left[P\left(\frac{q}{m};\, m\right) q - cq - \sum_{j=1}^{n} k_j q \right],$$

where $q = y_{\min}$ and $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$. By arguments similar to those in the proof of proposition 1, the weakly dominant strategy equilibrium is unique and equivalent to the smallest profit-maximizing monopoly output, $y_i^* = q^{\mathrm{M}}$ ($i = 1, \ldots, n$). Substituting for output gives total transfers as a function of output:

$$\sum_{i=1}^{n} t_i^*(q^{\mathrm{M}}).$$

By profit maximization

$$\left[P\left(\frac{q}{m};\, m\right) - c \right] q^{\mathrm{M}} > \sum_{j=1}^{n} k_j q^{\mathrm{M}}.$$

This implies that

$$m \sum_{i=1}^{n} t_i^* < \left[P\left(\frac{q^{\mathrm{M}}}{m};\, m\right) - c \right] q^{\mathrm{M}} = \rho^{\mathrm{M}} q^{\mathrm{M}} + m\Gamma^{\mathrm{M}}.$$

Q.E.D.

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