Measuring Benchmark Damages in Antitrust Litigation: Extensions and Practical Implications

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Abstract

This paper compares the "forecasting approach" and the "fully interacted approach" to estimation of cartel damages. We investigate the impact of relaxing the assumptions of exogeneity and stationarity, both theoretically and in Monte Carlo simulations. The results suggest that the advantages of the fully interacted approach are less clear and that the forecasting approach may be more robust to the relaxations of some of these maintained assumptions.

Keywords: Antitrust; Cartels; Damages estimation; Law and economics; Overcharges, Price-fixing

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1 Introduction

Quantifying cartel impact on prices is important for at least two reasons. From the standpoint of antitrust policy, understanding how effective cartels are in raising prices is instrumental in the design of optimal cartel sentences and fines. From the standpoint of private antitrust litigation, cartel overcharge is also critical, as it determines how much is at stake financially for both plaintiffs and defendants.

When quantifying cartel damages, two econometric approaches, both relying on the use of benchmark (or control) period, are quite popular and have been the subject of a small academic literature. These are the so-called dummy variable approach and the forecasting approach. In a recent paper, McCrary and Rubinfeld (2014) (hereafter MR), considered a third alternative, a fully interacted approach.[1] In their framework, the fully interacted approach and the forecasting approach produce identical but-for (counterfactual) prices for the cartel period. MR show that estimators of cartel overcharge based on these two approaches are consistent under different conditions.[2] They argue that the conditions for the consistency of the fully interacted model are more likely to hold in practice relative to those for the forecasting approach.

In this paper, we build upon MR and extend their original investigation in multiple directions. We study how the properties of these competing approaches change when some of the underlying assumptions are relaxed. Specifically, we consider the possibility of omitted variables and derive the conditions for estimation consistency. We also consider the case of integrated time series. In each of these extensions, we provide asymptotic results as well as Monte Carlo simulations to investigate finite-sample properties. The main takeaway is that the benefit of the fully interacted model is less clear than what MR's findings might have suggested. In fact, our findings imply that under certain conditions, the fully interacted approach can perform much worse than the corresponding forecasting approach.

There is a relatively small academic literature on these competing approaches. Nieberding (2006) provided an overview of the dummy-variable and the forecasting models; White, Marshall, and Kennedy (2006) discussed some empirical issues

[1] MR refer to the fully interacted model simply as the dummy variable model. For readers new to this literature and antitrust damages, it is important to know that this labeling is nonstandard. In antitrust litigation, the "dummy variable model" commonly refers to the model in which only the intercept is allowed to change during the conduct period, i.e., the "second dummy variable model" in MR. See, for example, Finkelstein and Levenbach (1983). For readers interested in why this understanding clarifies MR's discussion of the pros and cons of the forecasting and dummy variable approaches, see the working paper version of the article, available upon request.

[2] Note that in antitrust litigation, the term "overcharge" is typically used to refer to the percentage of cartel overcharge. In this paper, I follow MR to use the term "overcharge" to mean the total cartel overcharges in monetary terms.

of cartel damage estimation including the choice of predictors; Boswijk, Bun, and Schinkel (2019) examined the impact of misdating the start and the end of a cartel on cartel damages estimation in a framework similar to MR's.

The rest of the paper is organized as follows. In Section 2, we discuss the framework and briefly review MR's main results. In Section 3, we derive the conditions for consistency in the presence of omitted variables. Section 4 treats the case of nonstationary data. Selected simulation results are reported in Section 5 to corroborate the theoretical findings of the paper. Mathematical proofs are contained in a technical appendix and the complete set of simulation results are available upon request.

2 The theoretical framework

The econometric approaches to quantifying cartel damages depend on data availability. The situation we focus on in the paper is one in which a time series of the prices of the allegedly cartelized product is available and it covers both the alleged cartel conspiracy period, also known as the "conduct" period, as well as some pre- and/or postcartel period that provides an appropriate "benchmark" to estimate the cartel effects. The latter is often called simply the "benchmark" period in antitrust litigation. Figure 1 shows the Vitamin E price series in the In *re* Vitamins Antitrust Litigation as an example of this type of situation (Bernheim (2002)). Specifically, the price data are available in both the pre- and the post-cartel periods in this case.

MR are interested in estimating the expected (weighted) overcharges (or simply the antitrust damages):

$$OC^* = E(D_t Q_t \{Y_t(1) - Y_t(0)\}) = E(Q_t \{Y_t(1) - Y_t(0)\} | D_t = 1),$$

where D_t is an indicator variable taking on the value 1 during the conduct period and 0 during the non-conduct or the benchmark period; Q_t denotes the quantity sold in period t; $Y_t(1)$ is the actual prices during the conduct period and $Y_t(0)$ represents the counterfactual prices in the absence of the cartel conduct. Our starting point is a set of assumptions imposed by MR:

1. The DGP is $Y_t = \alpha + \beta' X_t + \delta D_t + \gamma' D_t X_t + \varepsilon_t$, where ε_t is mean zero and uncorrelated with X_t, D_t, and $D_t X_t$. Further, $\varepsilon_t = D_t u_t + (1 - D_t) v_t$ such that the error terms are allowed to be drawn from two different distributions between the conduct and non-conduct period.
2. (Y_t, X_t, D_t, Q_t) is a vector ergodic stationary process with existence of sufficient moments. And moments such as $\mathbf{E}[D_t Q_t Y_t]$ exist, and are finite and time invariant.

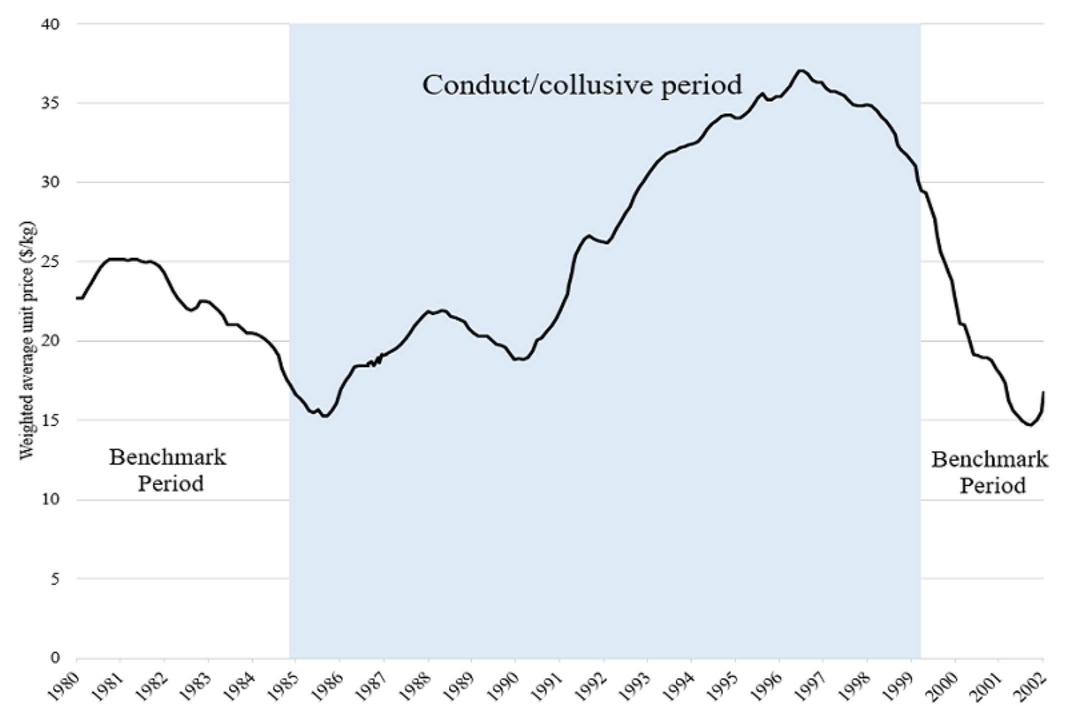

Figure 1: Vitamin E prices (digitized data based on Bernheim (2002), for illustration purposes only.)

3. Covariates X_t are not endogenous to the cartel conduct.
4. u_t and v_t are uncorrelated with D_t, i.e., $E[\varepsilon_t D_t] = 0$.
5. D_t is observable (i.e., the sample split between benchmark and conduct periods is known). [3]

For later reference, we also use T, T_c, and T_b to denote the size of the entire sample, the size of the conduct period, the size of the benchmark period, respectively. Following the literature on structural breaks, we also, for simplicity, assume that the conduct period goes from T_1 through T where $T_1 = [T\lambda]$, where $[.]$ is the integer part of the argument and $\lambda \in (0,1)$.

We also denote the estimated regression coefficients in the DGP by $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\delta}$, and $\widehat{\gamma}$. Note that in this fully interacted model, the same $\widehat{\alpha}$ and $\widehat{\beta}$ can be obtained if one regresses Y_t on an intercept and X_t using only the benchmark period data. With these notations, the "predicted" but-for prices during the conduct period are

[3] In the nomenclature of the structural break literature, this is the case of known breaks.

given by $\widehat{Y}_t(0) = \widehat{\alpha} + \widehat{\beta}' X_t$ and the fitted prices during the conduct period are simply $\widehat{Y}_t(1) = \widehat{\alpha} + \widehat{\delta} + (\widehat{\beta} + \widehat{\gamma})' X_t$.

In the forecasting approach, the overcharge is estimated by the difference between the actual conduct-period prices and the "predicted" but-for prices. The overcharge is formally given by,

$$\begin{aligned} \widehat{FC} &= \tfrac{1}{T_c} \textstyle\sum_{t \in Conduct} Q_t (Y_t - \widehat{Y}_t(0)) \\ &= \tfrac{1}{T_c} \textstyle\sum_{t \in Conduct} Q_t ((\alpha - \widehat{\alpha}) + (\beta - \widehat{\beta})' X_t + \delta + \gamma' X_t + u_t), \end{aligned}$$

In the fully interacted approach, the overcharge is estimated by the difference between the *fitted* conduct period prices and the "predicted" but-for prices as in the forecasting model, i.e.,

$$\begin{aligned} \widehat{OC_1} &= \tfrac{1}{T_c} \textstyle\sum_{t \in Conduct} Q_t (\widehat{Y}_t(1) - \widehat{Y}_t(0)) \\ &= \tfrac{1}{T_c} \textstyle\sum_{t \in Conduct} Q_t (\widehat{\delta} + \widehat{\gamma}' X_t), \end{aligned}$$

Consider the following assumptions regarding the covariances between the quantity Q_t and the error terms, where $C[A, B|D]$ is the covariance between random variables A and B conditional on D:

- Assumption 1: $C[u_t, Q_t | D_t = 1] = C[v_t, Q_t | D_t = 1] = 0$
- Assumption 2: $C[u_t, Q_t | D_t = 1] = C[v_t, Q_t | D_t = 1]$
- Assumption $1'$: $C[v_t, Q_t | D_t = 1] = 0$

MR show that the estimator of damages based on the fully interacted model is consistent under Assumption 1 or 2 whereas the estimator based on the forecasting approach is consistent under Assumption 1 or $1'$. Note that Assumptions $1'$ and 2 are not nested, so neither assumption can be said to be technically "stronger" or "weaker" than the other. MR argues there is a sense that Assumption $1'$ is stronger than Assumption 2, and hence the fully interacted model is likely a more robust approach in practice in their framework.

3 Allowing omitted variables in the conduct period

Consider a more general data generating process (DGP), where the factors driving prices during the benchmark (X_t) and the conduct periods (W_t) might be different, as a result of the cartel conduct:

$$\begin{aligned} Y_t(1) &= \alpha_2 + \beta_2 W_t + u_t \\ Y_t(0) &= \alpha_1 + \beta_1 X_t + v_t \\ W_t &= a_1 + a_2 X_t + e_t \end{aligned} \tag{1}$$

Hence, for simplicity and tractability, we allow the variables X_t and the omitted variables W_t to be linearly related. Note that by definition, $e_t \perp X_t$. We further assume that the econometrician, not aware of W_t, uses variables X_t for both the benchmark period and the cartel period in a fully interacted regression. The rest of MR's assumptions are maintained. This scenario is also arguably more plausible in practice than the "reversed" case in which the econometrician is aware of W_t instead of X_t. For example, MR (p. 68) suggested that one could select the variables based on the benchmark data and use them in a fully interacted regression.[4]

This is a plausible scenario. Economic theory implies that the effectiveness of a cartel may vary over time, depending on external factors (see, e.g., Green and Porter (1984), Rotemberg and Saloner (1986), and Ellison (1994)). So there is no guarantee that the very same factors in the benchmark period would drive the cartel prices simply with a different coefficient. Consider a case where the cartel members had to readjust or change their strategy at discrete points in time during the cartel period. Then W_t could be a piecewise linear function of X_t. That is, the cartel conduct could very well change the relationship between prices and the economic factors over time.[5]

Under this framework, the consistency conditions of the forecasting approach remain unchanged. However, analogous but different conditions are needed for the fully interacted estimator to be consistent. The following proposition states this formally.

Proposition 1 *In the model (1), the following condition needs to be satisfied for the estimator of damages based on the fully interacted approach to be consistent in MR's sense:* $C[u_t, Q_t | D_t = 1] + \beta_2 C[e_t, Q_t | D_t = 1] = C[v_t, Q_t | D_t = 1]$.[6]

[4] We distinguish this case from the situation where the covariates would be different even in the absence of the cartel conduct during the cartel period. In other words, we assume, as in MR, that in absence of the cartel conduct, the covariates would have been the same X_t across the entire sample. In actual applications, there may be evidence that would suggest a different set of covariates between the conduct and non-conduct periods for reasons other than the cartel conduct. Treating that case is beyond the scope of this paper.

[5] One can also postulate a more elaborate model in which the relationship between prices and the economic factors itself is a function of some other, potentially unobservable, factors.

[6] If W_t is a piecewise linear function of X_t, the only modification to the consistency condition in Proposition 1 is to replace e_t by a piecewise linear function of X_t. The proof is immediate and hence omitted.

This condition, effectively requiring a weighted sum of three covariances to be 0, is restrictive and unlikely to hold in practice. More importantly, it does not appear to be any more plausible than the condition under which the forecasting approach would produce a consistent estimate.[7] Unfortunately, because the estimation errors depend on β_2 and the covariance between e_t and Q_t, the relative direction of bias between the two approaches is unknown a priori. Note that it is easy to extend the results to allowing omitted variables in both the benchmark and the conduct periods.[8]

4 Nonstationary data

Especially in dynamic markets, prices and quantities can be nonstationary. For example, in In *re*: DRAM Antitrust Litigation, White (2007) and Marshall (2007) both argued and showed that the allegedly fixed DRAM prices were unit-root nonstationary. In fact, White (2007) identified cointegration relationships between the DRAM prices and the predictor variables in his application of the forecasting approach.

In this section, we allow the variables to have stochastic trends ($I(1)$). Specifically, we assume that

$$\begin{aligned} X_t &= d_x + \varepsilon_{x_t}, \\ \varepsilon_{x_t} &= \varepsilon_{x_{t-1}} + e_{x_t}, \end{aligned}$$

and

$$\begin{aligned} Q_t &= d_q + \varepsilon_{q_t}, \\ \varepsilon_{q_t} &= \rho \varepsilon_{q_{t-1}} + e_{q_t}, \end{aligned}$$

where d_x and d_q are constants. Extension to the case with deterministic time trend is beyond the scope of the paper, but is nevertheless straightforward. Next, the specification for Q_t does not rule out negative values if the disturbance term has the full support. This is unlikely to be a practical concern because the mean value d_q can be made sufficiently large.

Define $w_t' = \begin{pmatrix} e_{q_t} & e_{x_t}' & u_t & v_t \end{pmatrix}'$ and $S_{[Tr]} = \sum_{t=1}^{[Tr]} w_t'$. We require that the stochastic errors in the system satisfy the conditions so that a multivariate functional

[7] MR point out that in the special case of $v_t = u_t = \varepsilon_t$, Assumption 2 holds but $1'$ does not. With omitted variables, especially if the omitted variables are different during the conduct and benchmark periods, the condition $v_t = u_t = \varepsilon_t$ are highly unlikely to hold.

[8] See the working paper version of the paper for details.

central limit theorem holds for the partial sum process $S_{[Tr]}$. The technical appendix contains the detailed assumptions and our framework follows closely that of Park and Phillips (1988).

We consider the following two possibilities:

- The relationship between the covariates and the prices changes as a result of the cartel conduct, but they are still cointegrated.
- The cointegration breaks down as a result of the cartel conduct. This situation is possible when the cartel conduct is sufficiently "disruptive" so that prices fail to maintain the "equilibrium" relationship. Also note that whether cointegration holds or breaks down is empirically testable because it involves only observables (in particular, it does not involve the but-for or counterfactual prices).

4.1 If cointegration still holds

In this case, the fully interacted model during the conduct period is not a spurious regression in the classic sense (see Granger and Newbold 1974). The following proposition establishes the asymptotic behavior of the estimation errors.

Proposition 2 *Under the stated assumptions and if Q_t is $I(1)$ ($\rho = 1$),*

$$OC^* - \widehat{FC} \Rightarrow A - \Delta_{qv}.$$

And

$$OC^* - \widehat{OC^1} \quad \Rightarrow \quad B + \Delta_{qu} - \Delta_{qv},$$

where A and B are two mean zero $O_p(1)$ terms.[9] *Δ_{qu} and Δ_{qv} are the corresponding long-run covariance components associated with Q_t and u_t and Q_t and v_t, respectively. See the technical appendix for precise definitions of these terms. If Q_t is $I(0)$ ($|\rho| < 1$) instead, then*

$$OC^* - \widehat{FC} \quad \Rightarrow \quad -C[Q, v|D_t = 1].$$

And

$$OC^* - \widehat{OC^1} \quad \Rightarrow \quad C[Q, u|D_t = 1] - C[Q, v|D_t = 1].$$

[9] Note that while MR define OC^* as the expectation of the damages, which rules out situations where it is not well defined, we define OC^* more generally. See the technical appendix and the working paper version for more discussion.

If $Q_t = Q$ for all t,

$$OC^* - \widehat{FC} = O_p\left(\frac{1}{\sqrt{T}}\right)$$

and

$$OC^* - \widehat{OC^1} = O_p\left(\frac{1}{\sqrt{T}}\right).$$

In this case, when $Q = 1$, the results apply to unweighted overcharges.

Remark 1 *The terms Δ_{qu} and Δ_{qv} in the nonstationary case are extensions of $C[Q,u|D_t = 1]$ and $C[Q,v|D_t = 1]$ in the stationary case. Note that Δ_{qu} and Δ_{qv} depend on not only the contemporaneous correlations between the disturbance to Q_t and u_t/v_t, but also the serial correlations between them.*

Remark 2 *When Q_t is $I(0)$, our results show that the sufficient conditions for consistency in the stationary case also apply in the nonstationary case.*

Remark 3 *When Q_t is constant over time, the equivalence between the two approaches was proved in Proposition 1 of MR for the stationary case. Here we show that it extends to the nonstationary case and that the estimation errors converge to 0 at the rate of $\sqrt{T}$. Because unweighted and weighted overcharges become identical when quantity is constant over time, this result also implies that both approaches give consistent estimators when estimating unweighted or simple average overcharges.*

Remark 4 *Although we do not explicitly consider the case of omitted variables in the nonstationary case, we note that the theory extends easily. As long as the dependent variable (LHS) continues to be cointegrated with the included variables, then the stochastic orders of the estimation errors remain unchanged. The only necessary modification to the asymptotic distributions involves the covariances between Q_t and v_t and between Q_t and u_t. If the included variables are not cointegrated with the LHS during the conduct period, the theory in the next section applies.*

4.2 If cointegration breaks down

Now consider the case where the cointegration breaks down during the cartel period as a result of the cartel conduct. In this case, X_t is no longer cointegrated with Y_t. Instead, similar to DGP (3), we assume

$$
\begin{aligned}
Y_t(1) &= \alpha_2 + \beta_2 W_t + u_t \\
Y_t(0) &= \alpha_1 + \beta_1 X_t + v_t
\end{aligned} \tag{2}
$$

where $W_t = d_w + \varepsilon_{w_t}, \varepsilon_{w_t} = \varepsilon_{w_{t-1}} + e_{w_t}$, defined analogously as X_t. With the addition of W_t to the system, we also assume that the functional central limit theorem applies to the partial sum process defined above with the addition of e_{w_t}. Since the forecasting model is not affected by this alternative situation, we need examine only the fully interacted model here. The following proposition states the asymptotic property of the estimation error.

Proposition 3 *If Q_t is $I(1)$ ($\rho = 1$),*

$$
OC^* - \widehat{OC^1} = O_p(T),
$$

If Q_t is $I(0)$ ($|\rho| < 1$),

$$
OC^* - \widehat{OC^1} \Rightarrow O_p(1) + \gamma' \Delta_{wq} - \gamma' \Delta_{xq} + C[Q, u|D_t = 1] - C[Q, v|D_t = 1].
$$

Again, Δ_{wq} and Δ_{xq} are the corresponding long run covariance components associated with W_t and Q_t and X_t and Q_t, respectively.

If $Q_t = Q$ for all t,

$$
OC^* - \widehat{OC^1} = O_p\left(\frac{1}{\sqrt{T}}\right).
$$

Again, by setting $Q = 1$, this result applies to unweighted overcharges.

For the forecasting model, Proposition 3 continues to apply.

Remark 5 *If Q_t is $I(1)$, the estimation error of the fully interacted model diverges with order T, while the estimation error of the forecasting model is $O_p(1)$, as we have shown above. In a finite sample, however, the relative ranking of the bias and variance of the estimation errors of these approaches also depend on the size of Δ_{qv} and how close $\Delta_{qu} - \Delta_{qv}$ is to 0.*

Remark 6 *If Q_t is $I(0)$, the constant component of the estimation error contains not only $C[Q, u|D_t = 1] - C[Q, v|D_t = 1]$, but also $\gamma' \left(\Delta_{wq} - \Delta_{xq}\right)$. This is a particularly interesting result. It implies that both terms affect the bias of the fully interacted model. For example, one would need to impose the condition $\Delta_{wq} = \Delta_{xq}$, in addition to MR's assumption $C[Q, u|D_t = 1] = C[Q, v|D_t = 1]$, for the consistency of the fully interacted model.*

Remark 7 *When Q_t is constant over time (or when one is interested in unweighted overcharges), the two approaches become asymptotically equivalent and both give consistent estimators of damages. In fact, cointegration breakdown during the cartel period does not alter the asymptotic behavior of the fully interacted model. This may seem counterintuitive at first, because when cointegration breaks down, the fully interacted model would be a classic spurious regression during the cartel period. As is made clear in the proofs, however, the estimation error of the fully interacted approach is equal to the sum of the estimation error of the forecasting approach and an extra term* $\frac{1}{T_c}\sum_{t=T_1}^{T} Q_t R_t(1)$, *where* $R_t(1)$ *is the OLS regression residuals during the cartel period. When* Q_t *is constant,* $\frac{1}{T_c}\sum_{t=T_1}^{T} Q_t R_t(1) = \frac{1}{T_c} Q \sum_{t=T_1}^{T} R_t(1) = 0$, *where the last equality is due to the orthogonality condition of the OLS. This is the same reason for the equivalence result in MR's Proposition 1.*

These results show that, unless Q_t is constant over time, the behavior of the fully interacted model can be significantly affected by the cointegration breakdown. These theoretical results suggest that to be confident that the fully interacted model improves upon the forecasting model, it is important to analyze the conduct period as carefully as the benchmark period. Even so, our results show that the behavior of the forecast errors is complex enough that it is unclear which approach is better in practice.

5 Monte Carlo Simulations

5.1 Introduction

Our analysis yields a number of interesting implications about the forecasting approach and the fully interacted approach. In this section, we report a selected set of simulation results to illustrate some of the implications. A more comprehensive set of simulation results are available upon request. Specifically, we report the following simulations:

- With stationary data, the MSE of these approaches depend on a number of factors. Even when the fully interacted estimator is consistent or unbiased, the estimator based on the forecasting approach can be better in the MSE sense.[10]
- When the data are nonstationary, we examine the following implications:

[10] For a formal discussion, see the working paper version of the paper.

- When Q_t is $I(1)$ and the cointegration relationship breaks down in the conduct period, the bias of the fully interacted model grows with the sample size. But in a finite sample, its performance also depends on the magnitude of Δ_{qv} and Δ_{qu}.

Following MR, the size of our simulated sample is 100 unless otherwise noted. This sample size is consistent with what is often observed in antitrust litigation (roughly nine years of monthly observations) and also facilitates a comparison with MR's results.

5.2 Simulation setup

We follow closely MR's simulation setup. In each simulation, the start date is a random draw from a binomial distribution with parameters 100 and $\frac{2}{3}$. This implies that the alleged cartel period starts at roughly two-thirds of the sample. Specifically, The covariate X_t is generated according to[11]

$$X_t = 1 - 0.015t + 0.25X_{t-1} + e_t.$$

The variable is initiated at $X_0 = 0$ and e_t is iid standard normal. The "price" variable is generated according to

$$Y_t = 10 + 2X_t + 4D_t + \gamma D_t X_t + \varepsilon_t,$$

where $\varepsilon_t = D_t Z_t \tilde{u}_t + (1 - D_t) Z_t \tilde{v}_t$ and $\tilde{u}_t$, $\tilde{v}_t$, and Z_t are i.i.d. standard normal and independent from each other and e_t. The coefficients are chosen arbitrarily. Finally, the quantity Q_t is either set to be 150,000 or generated by an $AR(1)$ model, as in

$$Q_t = 75 + 0.5Q_{t-1} + v_u u_t + v_v v_t + \varepsilon_t.$$

Again, $Q_0 = 0$ and ε_t is also i.i.d. standard normal and independent from ε_t and e_t.
MR considered six cases, summarized in the table below. We focus on the fully interacted and the forecasting approaches here.

5.3 Comparing RMSE

In MR's simulation setting, intuition suggests that two sets of parameters affect the relevant performance of the two approaches. First, everything else the same,

[11] It is interesting to note that the simulated variables in MR's Monte Carlo experiments in fact are not stationary.

Table 1: MR's simulation setup

Model	Model for Q_t	v_u	v_v	γ	Consistent estimators
1	150,000	-	-	0	$\widehat{FC}$, $\widehat{OC_1}$
2	150,000	-	-	1	$\widehat{FC}$, $\widehat{OC_1}$
3	AR(1)	0	0	1	$\widehat{FC}$, $\widehat{OC_1}$
4	AR(1)	3	3	1	$\widehat{OC_1}$
5	AR(1)	3	0	1	$\widehat{FC}$
6	AR(1)	0	3	1	None

the smaller v_u and v_v are, the closer the performance of forecasting approach is to the fully interacted model. Second, everything else the same, one expects the fully interacted model to have greater variability when either the benchmark or the conduct period is short (see Proposition 2).

To demonstrate this possibility, we focus on MR's Model 4 and make the following modifications to the simulation setup. We consider smaller values of v_u and v_v and other values of t^*, including 15 and 85.[12] The former case corresponds to the scenario where the benchmark or the pre-conduct period is quite short and the latter case to the scenario where the conduct period is quite short. Table 2 shows the mean, the standard deviation, and the RMSE for a variety of parameters.

The results confirm our intuition regarding the relative performance in terms of MSE. When v_u and v_v are relatively large, the RMSE of the forecasting approach are consistently larger, as is shown by MR. However, when both v_u and v_v are equal to 0.5, and the forecasting approach has smaller RMSE for all three possible values of t^*. In an unreported simulation, we also considered a range of values for v_u in the context of Model 5. The results show that the forecasting model is always better in terms of MSE.

5.4 Nonstationary data

We next turn to the case of nonstationary data. Specifically, we simulate the covariate X_t according to a simple random walk plus noise DGP,

$$X_t \quad = \quad 1 + \varepsilon_{x_t},$$

[12] We also remove the randomness of t^* as a result. So t^* are fixed at 15, 66, and 85 in our experiments. This change allows us to focus on other sampling variations of the data, rather than the start or the length of the cartel conduct period.

Table 2: The effect of estimation sample size on finite sample performance

t^*				15		66		85	
Model	v_u	v_v		OC_1	FC	OC_1	FC	OC_1	FC
4	0.5	0.5	*bias*	-7.43	-48.77	12.57	-3.35	-0.39	-6.96
			std	(5,384.57)	(5,384.16)	(1,267.58)	(1,267.39)	(701.31)	(701.25)
			rmse	5,384.31	5,384.12	1,267.58	1,267.34	701.27	701.24
	1	1	*bias*	-9.16	-91.93	10.91	-20.99	-1.85	-15.01
			std	(5,384.93)	(5,384.41)	(1,267.56)	(1,267.27)	(701.36)	(701.24)
			rmse	5,384.66	5,384.92	1,267.55	1,267.39	701.33	701.37
	2	2	*bias*	-12.62	-178.24	7.60	-56.29	-4.77	-31.11
			std	(5,385.81)	(5,385.06)	(1,267.79)	(1,267.29)	(701.68)	(701.45)
			rmse	5,385.56	5,387.74	1,267.75	1,268.48	701.66	702.10
	3	3	*bias*	-16.08	-264.56	4.28	-91.58	-7.69	-47.20
			std	(5,386.93)	(5,385.93)	(1,268.38)	(1,267.65)	(702.28)	(701.93)
			rmse	5,386.69	5,392.16	1,268.32	1,270.89	702.28	703.48
	4	4	*bias*	-19.54	-350.88	0.97	-126.87	-10.61	-63.30
			std	(5,388.29)	(5,387.03)	(1,269.31)	(1,268.35)	(703.15)	(702.68)
			rmse	5,388.06	5,398.18	1,269.24	1,274.62	703.19	705.49
	5	5	*bias*	-23.00	-437.20	-2.35	-162.17	-13.53	-79.40
			std	(5,389.88)	(5,388.35)	(1,270.59)	(1,269.40)	(704.29)	(703.71)
			rmse	5,389.66	5,405.79	1,270.53	1,279.65	704.38	708.14
	6	6	*bias*	-26.46	-523.51	-5.66	-197.46	-16.45	-95.50
			std	(5,391.71)	(5,389.89)	(1,272.22)	(1,270.78)	(705.70)	(705.01)
			rmse	5,391.51	5,414.99	1,272.17	1,285.97	705.86	711.41

Note: In thousands of dollars.

$$\varepsilon_{x_t} = \varepsilon_{x_{t-1}} + e_{x_t},$$

where e_{x_t} is i.i.d. standard normal. When we study the case of cointegration breakdown, we simulate another random walk plus noise process W_t independent of X_t:

$$W_t = 1 + \varepsilon_{w_t},$$
$$\varepsilon_{w_t} = \varepsilon_{w_{t-1}} + e_{w_t}.$$

Again, we set e_{x_t} to be i.i.d. standard normal. And in this latter case, we assume Y_t is a function of W_t, instead of X_t, during the conduct period. Otherwise, the functional form of the DGP for Y_t remains the same. The results for Model 1 through Model 3 and Model 6 are available upon request. To save space, we do not report them here. But as a general matter, the two approaches perform similarly for those models.

5.4.1 Cointegration breaks down, Q is I(1)

As we can see in Table 3, when Q_t is nonstationary, the fully interacted model, consisting of a spurious regression for the cartel period, performs worse than the

Table 3: Cointegration breaks down and Q_t is $I(1)$

			t^*	15		66		85	
Model	v_u	v_v		OC_1	FC	OC_1	FC	OC_1	FC
4	0.5	0.5	*bias*	27.54	-17.46	3.46	-6.20	-13.95	-16.43
			std	(17,243.23)	(17,156.53)	(1,716.97)	(1,687.93)	(785.18)	(781.57)
			rmse	17,242.39	17,155.68	1,716.89	1,687.85	785.26	781.70
	1	1	*bias*	44.94	-15.77	9.15	-6.26	-11.58	-16.87
			std	(17,373.17)	(17,177.99)	(1,748.40)	(1,692.16)	(791.03)	(784.00)
			rmse	17,372.36	17,177.14	1,748.34	1,692.08	791.07	784.14
	2	2	*bias*	79.75	-12.39	20.54	-6.36	-6.83	-17.75
			std	(17,905.36)	(17,283.94)	(1,873.20)	(1,709.61)	(813.25)	(793.62)
			rmse	17,904.64	17,283.08	1,873.22	1,709.53	813.24	793.78
	3	3	*bias*	114.55	-9.00	31.93	-6.46	-2.09	-18.62
			std	(18,772.72)	(17,472.52)	(2,067.12)	(1,738.73)	(848.52)	(809.39)
			rmse	18,772.13	17,471.65	2,067.27	1,738.65	848.48	809.56
	4	4	*bias*	149.36	-5.62	43.32	-6.56	2.66	-19.49
			std	(19,931.53)	(17,741.11)	(2,312.86)	(1,778.94)	(895.28)	(830.95)
			rmse	19,931.09	17,740.23	2,313.15	1,778.86	895.24	831.14
	5	5	*bias*	184.17	-2.24	54.71	-6.66	7.40	-20.37
			std	(21,334.37)	(18,086.15)	(2,595.72)	(1,829.51)	(951.85)	(857.86)
			rmse	21,334.09	18,085.24	2,596.17	1,829.43	951.83	858.06
	6	6	*bias*	218.98	1.14	66.10	-6.77	12.15	-21.24
			std	(22,936.49)	(18,503.35)	(2,904.90)	(1,889.62)	(1016.60)	(889.64)
			rmse	22,936.39	18,502.42	2,905.50	1,889.54	1,016.62	889.85
5	0.5	0	*bias*	23.29	-20.43	6.18	-5.54	-13.69	-16.16
			std	(17,243.10)	(17,169.45)	(1,713.66)	(1,687.64)	(784.03)	(781.39)
			rmse	17,242.26	17,168.60	1,713.59	1,687.56	784.11	781.52
	1	0	*bias*	36.45	-21.70	14.60	-4.93	-11.06	-16.32
			std	(17,326.30)	(17,193.58)	(1,730.86)	(1,690.08)	(786.94)	(782.85)
			rmse	17,325.47	17,192.74	1,730.84	1,690.00	786.98	782.98
	2	0	*bias*	62.77	-24.25	31.43	-3.71	-5.80	-16.64
			std	(17,631.76)	(17,274.37)	(1,798.20)	(1,699.54)	(798.19)	(788.19)
			rmse	17,630.99	17,273.52	1,798.39	1,699.46	798.17	788.33
	3	0	*bias*	89.09	-26.81	48.26	-2.48	-0.54	-16.96
			std	(18,114.22)	(17,397.97)	(1,905.26)	(1,715.03)	(816.41)	(796.70)
			rmse	18,113.53	17,397.12	1,905.78	1,714.94	816.37	796.84
	4	0	*bias*	115.41	-29.36	65.09	-1.26	4.72	-17.28
			std	(18,760.03)	(17,563.49)	(2,045.82)	(1,736.37)	(841.15)	(808.27)
			rmse	18,759.45	17,562.63	2,046.75	1,736.28	841.12	808.42
	5	0	*bias*	141.73	-31.91	81.93	-0.04	9.98	-17.60
			std	(19,553.02)	(17,769.74)	(2,213.50)	(1,763.35)	(871.85)	(822.78)
			rmse	19,552.56	17,768.88	2,214.91	1,763.26	871.86	822.93
	6	0	*bias*	168.05	-34.46	98.76	1.19	15.25	-17.92
			std	(20,476.10)	(18,015.34)	(2,402.63)	(1,795.72)	(907.90)	(840.08)
			rmse	20,475.76	18,014.48	2,404.54	1,795.63	907.98	840.23

Note: In thousands of dollars.

forecasting approach in terms of the RMSE for all cases considered. There are a few cases in which the fully interacted approach has smaller biases ($t^* = 85$ and Model 5). However, when we increase the sample size to 200, the forecasting approach performs better uniformly across the cases considered, consistent with the implication of our asymptotic theory. These results are reported in Table 4.

6 Conclusions

In this paper, we studied how some practically plausible conditions affect the performance of the fully interacted approach and the forecasting approach effects. In short, our findings suggest that the relative advantages of the fully interacted model are less clear. In fact, there are several factors to consider before choosing the fully interacted model over a standard forecasting model. First, because the fully interacted model uses both the benchmark and conduct period data (to separately fit regressions), one needs to ask whether there is sufficient data variation in both time periods. While lack of data variation during the benchmark period presents challenges to both the fully interacted and the forecasting approaches, lack of data variation during the conduct period affects only the former approach. Second, allowing omitted variables, especially allowing different omitted variables in the conduct and benchmark periods reveals that the consistency conditions for the fully interacted approach do not make it empirically any more plausible than the forecasting approach. Third, if the prices and covariates are nonstationary, is there a cointegration relationship between the covariates and the prices throughout the sample? As shown in this paper, the fully interacted approach is most vulnerable when the cointegration breaks down during the conduct period. All these factors require a careful look at both the conduct and benchmark data. In contrast, the forecasting approach requires a close examination of only the benchmark data. And practitioners should also keep in mind that both approaches give identical but-for (counterfactual) prices.

As noted in the Introduction, this paper leaves out some interesting topics that could further our understanding of the differences between the two approaches. In addition to those mentioned earlier, another worthy research effort is to extend the results in the present paper to the more challenging case of autoregressive regression models, especially when the time series are nonstationary.

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Table 4: Cointegration breaks down and Q_t is $I(1)$, $T = 200$

			t^*	15		66		85	
Model	v_u	v_v		OC_1	FC	OC_1	FC	OC_1	FC
4	0.5	0.5	*bias*	355.35	321.17	76.49	63.12	62.63	47.00
			std	(56,894.18)	(56,129.75)	(9,475.36)	(8,115.83)	(6589.76)	(5526.52)
			rmse	56,892.44	56,127.86	9,475.19	8,115.67	6,589.73	5,526.44
	1	1	*bias*	348.15	278.97	100.32	64.98	86.82	43.79
			std	(57,814.76)	(56,343.83)	(10,642.07)	(8,158.83)	(7524.05)	(5550.82)
			rmse	57,812.92	56,341.70	10,642.01	8,158.68	7,524.18	5,550.72
	2	2	*bias*	333.75	194.59	147.98	68.70	135.20	37.39
			std	(61,338.23)	(57,178.43)	(14,439.56)	(8,313.14)	(10492.98)	(5649.67)
			rmse	61,336.07	57,175.90	14,439.60	8,313.01	10,493.33	5,649.51
	3	3	*bias*	319.36	110.20	195.64	72.42	183.58	30.98
			std	(66,786.13)	(58,534.07)	(19,187.35)	(8,554.11)	(14133.35)	(5812.42)
			rmse	66,783.55	58,531.25	19,187.39	8,553.98	14,133.83	5,812.21
	4	4	*bias*	304.97	25.81	243.30	76.14	231.95	24.57
			std	(73,733.11)	(60,375.67)	(24,335.45)	(8,874.68)	(18043.23)	(6033.91)
			rmse	73,730.06	60,372.66	24,335.45	8,874.56	18,043.82	6,033.66
	5	5	*bias*	290.57	-58.57	290.96	79.85	280.33	18.16
			std	(81,798.13)	(62,660.39)	(29,676.26)	(9,266.60)	(22079.92)	(6307.95)
			rmse	81,794.56	62,657.28	29,676.20	9,266.48	22,080.59	6,307.66
	6	6	*bias*	276.18	-142.96	338.62	83.57	328.71	11.76
			std	(90,683.37)	(65,341.76)	(35,121.97)	(9,721.24)	(26184.82)	(6628.02)
			rmse	90,679.26	65,338.65	35,121.85	9,721.12	26,185.57	6,627.70
5	0.5	0	*bias*	378.65	338.12	91.57	66.14	71.02	52.17
			std	(56,759.45)	(56,107.18)	(9,260.33)	(8,112.53)	(6413.93)	(5523.51)
			rmse	56,757.88	56,105.40	9,260.32	8,112.40	6,414.00	5,523.48
	1	0	*bias*	394.75	312.88	130.48	71.02	103.59	54.14
			std	(57,242.50)	(56,224.66)	(9,852.83)	(8,140.05)	(6890.16)	(5535.88)
			rmse	57,240.99	56,222.72	9,853.20	8,139.95	6,890.59	5,535.86
	2	0	*bias*	426.96	262.41	208.30	80.77	168.74	58.08
			std	(59,074.66)	(56,649.33)	(11,986.56)	(8,228.00)	(8577.42)	(5584.82)
			rmse	59,073.25	56,647.11	11,987.77	8,227.98	8,578.65	5,584.84
	3	0	*bias*	459.16	211.93	286.11	90.52	233.88	62.02
			std	(61,969.57)	(57,321.93)	(14,906.74)	(8,358.59)	(10843.02)	(5665.31)
			rmse	61,968.17	57,319.46	14,908.74	8,358.66	10,845.00	5,665.36
	4	0	*bias*	491.37	161.45	363.93	100.27	299.02	65.96
			std	(65,787.09)	(58,233.86)	(18,239.47)	(8,529.87)	(13396.70)	(5776.01)
			rmse	65,785.64	58,231.18	18,242.19	8,530.03	13,399.37	5,776.10
	5	0	*bias*	523.57	110.98	441.75	110.02	364.17	69.90
			std	(70,377.24)	(59,374.10)	(21,796.32)	(8,739.45)	(16101.97)	(5915.24)
			rmse	70,375.67	59,371.24	21,799.71	8,739.70	16,105.28	5,915.36
	6	0	*bias*	555.78	60.50	519.57	119.77	429.31	73.84
			std	(75,599.41)	(60,729.80)	(25,483.62)	(8,984.64)	(18893.82)	(6081.04)
			rmse	75,597.68	60,726.79	25,487.64	8,984.99	18,897.75	6,081.18

Note: In thousands of dollars.

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