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Summary

The begin and end dates of cartels are often ambiguous, despite competition authorities stating them with precision. The legally established infringement period(s) from documentary evidence need not coincide with the period(s) of actual cartel effects. In this paper, we show that misdating cartel effects leads to a (weak) overestimation of but-for prices and an underestimation of overcharges. Total overcharges based on comparing but-for prices to *actual* prices are a (weak) underestimation of the true amount overcharged, irrespective of the type and size of the misdating. The bias in antitrust damage estimation based on *predicted* cartel prices can have either sign. We extend the before-during-and-after method with an empirical cartel dating procedure, which infers structural breaks of unknown number and dates that mark the actual begin and end dates of the collusive effects. Empirical findings in the European *Sodium Chlorate* cartel corroborate our theoretical results.

Keywords: Cartel, antitrust damages, dates, structural change, break test, but-for.

1 Introduction

Collusive practices such as price-fixing and market sharing allow firms to exert market power they would otherwise not have, artificially restrict output and increase prices. Cartels are per se illegal and when caught can expect fines and liability for antitrust damages. To determine the quantum of these requires knowing with some precision when and how long the cartel was effective. In their prosecutions of cartel offenses, antitrust agencies specify

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the begin and end dates of cartel violations with seeming precision, often to the day. These formal cartel dates are typically based on documentary evidence of communication, such as minutes of the initiating cartel meeting or e-mails in which members disassociate themselves from the agreements. The infringement period is subject also to (settlement) negotiations.[1]

The formal cartel dates need not coincide with the point(s) in time at which the anti-competitive effects produced by the cartel violation began and ended either. The collusive agreements may take a while to take effect, become ineffective before its members ultimately disband, or rather have lasting effects—including unilateral incentives for former cartellists to keep post-cartel prices up in order to mask their conspiracy.[2] Also, between periods of coordinated high prices, the cartel spell can include temporary price wars or reversionary episodes due to internal tensions, from which the cartel regathered.[3]

In this paper, we study the consequences for antitrust damage estimation of using cartel dates that are different from the actual begin and end dates of the period(s) in which the collusion had effects. Obviously, different dates imply different periods and volumes purchased over which damages can be claimed. We show that misdating the period(s) in which a cartel was effective introduces a bias that leads to a (weak) overestimation of but-for prices and an underestimation of overcharges. The effect on total damages depends crucially on whether overcharges are based on predicted or actual cartel prices. When the effective cartel dates are *a priori* unknown, the common approach of comparing but-for prices to predicted values is shown to be unreliable. The use of observed prices, on the other hand, leads to a (weak) underestimation of cartel damages, irrespective of the type and size of the misdating error.

To overcome bias in damage estimation due to incorrect cartel dates, we propose an empirical procedure for determining the number and dates of the effective cartel periods in price time-series that is based on econometric tests for multiple structural change as developed by Bai and Perron (1998, 2003). The method is applied to the European *Sodium Chlorate* cartel. While the cartel became effective shortly after its formal begin date, we find that it was a single period with overcharges that lasted long after the formal cartel end date. The empirical findings corroborate our theoretical results. The actual total cartel damage in *Sodium Chlorate* is underestimated by more than 25% when using the formal dates.

[1] American Bar Association (2014), *Econometrics: Legal, Practical, and Technical Issues*, 2nd edition, page 318; Commission of the European Communities (2013), *Practical Guide, Quantifying harm in actions for damages based on breaches of Article 101 or 102 of the Treaty of the Functioning of the European Union*, recital 43 and footnote 38. Both call for the development of econometric techniques to determine effective cartel dates.

[2] See Harrington (2004).

[3] See Porter (1983), Green and Porter (1984), Rotemberg and Saloner (1986) and Ellison (1994).

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Even though most cartel damage studies disclosed rely on formal cartel dates, cartel periods are subject of dispute. In the few public cases in which the effective cartel dates are argued, methods vary. White (2001) concludes a shorter effective period of the *Lysine Cartel* than alleged by inspection—as well as a higher but-for price. Bernheim (2008) and Marshall et al. (2008) observe that the *Vitamins Cartel* potentially affected prices beyond the formal cartel periods and analyze a number of alternative effective dates. David and Garces (2010) mention that the start and end dates of cartel effects may be ambiguous, but offer no methods for determining them. Hüschelrath et al. (2013) model the transition from cartel to non-cartel regime in the German cement cartel with dummy fractions. Boshoff and van Jaarsveld (2018) apply a Markov regime-switching model to analyze recurrent cartel overcharges in the South African cement market.

While structural break tests are widely used in economics, published applications in determining cartel effects are few. Carlton and Leonard (2004) report on an application of Bai-Perron tests on a plaintiff's price-series in an undisclosed case in which the authors acted for the defense. Having found breaks at all in only a small percentage of runs of Monte Carlo simulations, the authors conclude no statistically significant effect from the alleged conspiracy, and assert that the plaintiffs had determined their damages period by "cherry picking" the time-series for price declines. For the purpose of detecting suspicion of collusion, Harrington (2008) proposes the Quandt (1960)-Andrews (1993) test for a single unknown break date. Crede (2015) explores the Bai-Perron tests also as part of an empirical cartel screen.

This paper is organized as follows. In Section 2 we review common time-series regression analysis to estimate but-for prices and overcharges. We analyze in detail under which assumptions unbiased cartel damage estimation occurs. Section 3 sets out how using formal cartel dates that are different from the effective cartel dates affects the damage estimates. In Section 4 we describe the empirical cartel dating procedure. Section 5 illustrates the effects of misdating bias by the European *Sodium Chlorate* cartel. Section 6 concludes. The proofs and supportive simulation results are given in the online appendix.

2 Estimating Cartel Damages using Time Series

Statistical inferences on antitrust damages in empirical cartel studies often rely on time-series analysis, comparing prices over time on the same market, where outside the alleged effective cartel period(s) the industry is assumed to have been in its normal form of competition. Figure 1 is an illustration of the estimation of cartel effects in a hypothetical case in the European Commission's 2013 *Practical Guide* (recital 79) on quantifying harm in actions for antitrust damages.

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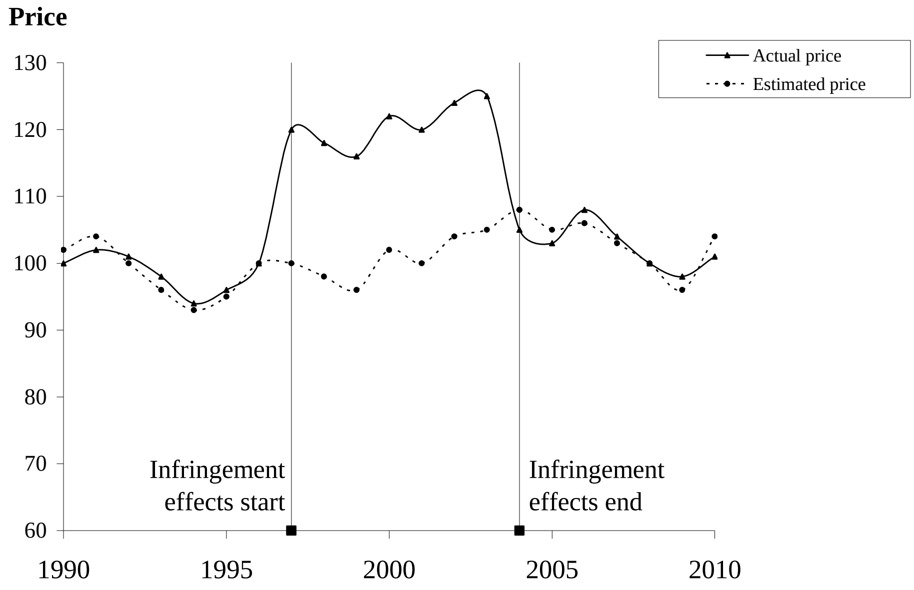

Figure 1: Cartel price effects in the *Practical Guide*

The solid line (labeled Actual price) is distinctly elevated during a single effective cartel spell between the marked start and end of the infringement effect. The dotted line (Estimated price) is what would have been prices under competition. Typically these but-for prices are estimated using multivariate regression models that include the relevant factors affecting product prices. The difference between the cartel price and estimated but-for price is the price overcharge by the cartel. A claimant's total amount overcharged is the sum over all periods in which the cartel was effective, of each period's price overcharge multiplied with the total quantity purchased during that period.

For determining but-for prices, two regression approaches are presented. In the forecasting approach, competitive prices during the periods of cartel effects are estimated only from data outside those periods. In case of a long lasting cartel and a short post-cartel period, data may be insufficient to forecast. The alternative before-during-and-after method also involves data from during the cartel periods by including a dummy variable in the regression that quantifies the cartel occurrence as a price shift. While this potentially increases the accuracy of the estimates, the underlying assumption that coefficients are constant over the damage and the benchmark period(s) may not hold. For example, cost pass-through can be structurally lower under collusion than competition. Also, the use of explanatory variables that may have been affected by the cartel, such as capacity, should be avoided.[4]

[4]See Nieberding (2006), White et al. (2006), Godek (2011), Marshall and Marx (2012) and McCrary and Rubinfeld (2014) for further discussion on the relative merits of both approaches.

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Many recent cartel studies make use of dynamic regression models to quantify cartel overcharges.[5] It is natural and advantageous to include lagged prices as additional explanatory variables. Parametrically modelling short-run dynamics allows for the typical gradual price adjustments over multiple periods and transition of non-cartel to cartel states. Cartel talks often aim at maintaining or increasing prices relative to recent values leading to serial correlation in prices. Also the lagged price terms contain valuable information on relevant other, but unobserved price determinants.

In this paper we take the dummy variable approach to a dynamic regression model. Suppose that price over time develops according to the data generating process (DGP)

$$p_t = \alpha_1 + \alpha_2 D_t + \beta' x_t + \gamma p_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{1}$$

where p_t is the product unit price in period t, D_t the dummy variable quantifying the presence of the cartel effect, x_t a set of explanatory variables, p_{t-1} the lagged value of price and ε_t an error term. Collusion is aimed at increasing prices, i.e. $\alpha_2 > 0$. The coefficient α_2 measures the immediate or short-run response of the price to the collusion as a price level shift. Typical explanatory variables x_t that control for regular changes in prices include cost factors, and demand and supply shifters. The regressor p_{t-1} captures the partial adjustment in prices due to recent changes in the cost structure and market conditions. We assume $0 < \gamma < 1$, i.e. the dynamic relation is stable. Note that DGP (1) is a general reduced form equation of the price that encompasses various specifications with (and without) autoregressive distributed lags.[6]

In DGP (1), the occurence of the cartel is specified as an intercept shifter, but the model can be generalized to account for slope shifters too. For example, the autoregressive coefficient γ can be different between cartel and non-cartel regimes when the cartel is particularly aimed at increasing prices relative to recent values. Furthermore, a different cost pass-through by a cartel implies changing elements of β. We find no empirical evidence in our analysis of the *Sodium Chlorate* cartel that suggests a need to include slope shifters, and therefore maintain constant slope coefficients as in (1).

We allow for recurrent collusion that alternates with any number of intermittent episodes of low prices, where the cartel may go through price wars or temporary break-ups. These are assumed to be Nash-reversions to competition by the use of a single cartel dummy. That is, the sample period $\mathbb{T} = \{1, \ldots, T\}$ consists of periods with and without cartel effects,

[5] See White et al. (2006), Nieberding (2006), Roeller and Steen (2006), Bernheim (2008) and David and Garces (2010).

[6] Including the specifications used in the earlier empirical cartel studies referred to in the previous two footnotes. Without loss of generality, we consider the case of one lagged value of the dependent variable regressor only. The vector of explanatory variables x_t can include both contemporaneous and lagged regressors.

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labeled $\mathbb{T}_C$ and $\mathbb{T}_N$ respectively, with $\mathbb{T} = \mathbb{T}_C \cup \mathbb{T}_N$. We denote the number of periods in $\mathbb{T}_C$ and $\mathbb{T}_N$ with T_C and T_N respectively, with $T_C + T_N = T$. D_t is 1 in effective cartel periods ($t \in \mathbb{T}_C$) and 0 otherwise ($t \in \mathbb{T}_N$). In case of recurrent collusion, D_t defines $R > 1$ periods of cartel effects, i.e. $\mathbb{T}_C = \mathbb{T}_{C_1} \cup \ldots \cup \mathbb{T}_{C_R}$.

The standard before-during-after scenario of a single and continuous period of collusive high prices ($R = 1$) as in Figure 1, which we will analyze in Section 3.3 for application to *Sodium Chlorate*, has an actual begin date T_B and end date T_E, with $1 < T_B < T_E < T$, so that $\mathbb{T}_C = \{T_B + 1, \ldots, T_E\}$ and $\mathbb{T}_N = \{1, \ldots, T_B, T_E + 1, \ldots, T\}$ and:

$$D_t = \begin{cases} 0, & t \leq T_B, \\ 1, & T_B < t \leq T_E, \\ 0, & t > T_E. \end{cases} \tag{2}$$

The but-for price in period t is the expected price level in t when the cartel dummy in (1) is switched off ($D_t = 0$), or

$$bfp_t = \alpha_1 + \beta' x_t + \gamma bfp_{t-1} + \varepsilon_t, \quad t \in \mathbb{T}_C. \tag{3}$$

Outside effective cartel periods, the but-for price and actual price coincide. Within effective cartel periods, the actual price minus the but-for price in each period is that period's overcharge

$$O_t = p_t - bfp_t, \quad t \in \mathbb{T}_C. \tag{4}$$

Typically, the basis for a cartel damages claim is the total amount overcharged, defined (before interest) as

$$CD = \sum_{t \in \mathbb{T}_C} O_t Q_t, \tag{5}$$

in which Q_t is the actual quantity purchased in period t.

We analyze whether population magnitudes such as the but-for price (3), overcharge (4) and total cartel damages (5) are accurately estimated based on a sample of time-series observations (p_t, x_t', D_t), combined with per period quantities purchased Q_t, $t = 1, \ldots, T$. The possible bias in these estimators can be characterized in terms of their probability limit, which is the value around which their distribution is increasingly concentrated as the sample size increases. In specific situations, the probability limit will coincide with the mean of the estimators. However, in general this expected value cannot be derived analytically. For its approximation by the probability limit to be well defined, we will consider the estimation of averages per period—rather than the total amount, which does not converge as T increases. In the following, we assume that as the sample size increases, the number of observations in both the cartel and the non-cartel period (T_C and T_N) increases proportionally, which is

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customary in the econometric literature on structural change.[7] Similarly, we assume that the length of each of the given R effective cartel periods increases proportionally with the sample size.

The probability limit of the average overcharge over the cartel period(s)

$$\bar{O} = \frac{1}{T_C} \sum_{t \in \mathbb{T}_C} O_t, \tag{6}$$

can be shown to be equal to

$$\text{plim}\, \bar{O} = \frac{\alpha_2}{1-\gamma}. \tag{7}$$

That is, DGP (1) implies an immediate shift of magnitude α_2 in the level of the product price as a result of the collusive agreement, controlling for usual developments in price caused by the explanatory variables in x_t. Due to the autoregressive short-run dynamics, this initial effect accumulates by gradual price adjustment over time to the full cartel effect of size $\alpha_2/(1-\gamma) > 0$.

The effects of the various explanatory variables on the price in model (1) are estimated by the standard Ordinary Least Squares (OLS) method. Consistent OLS estimation of the regression coefficients is feasible under the following assumption.

Assumption 1 *DGP (1) satisfies:*

$$\begin{aligned} E\left[\varepsilon_t | \mathcal{F}_t\right] &= 0, \\ \text{Var}\left(\varepsilon_t | \mathcal{F}_t\right) &= \sigma_N^2 + D_t\left(\sigma_C^2 - \sigma_N^2\right), \end{aligned}$$

where $\mathcal{F}_t = \{D_{t-j}, x_{t-j}, p_{t-j-1}, j = 0, 1, \ldots\}$*; and*

$$\text{Cov}(D_t, x_s) = 0, \qquad \forall s, t.$$

Furthermore, the processes p_t and x_t are jointly stationary within the cartel and non-cartel regimes.

For consistency, it is only required that the contemporaneous error term ε_t is uncorrelated with current values of D_t and x_t. Lagged feedback of price to its determinants is allowed for.[8] The begin of a cartel period, for example, need not be exogenous but may be induced by a recent price fall. The assumption also implies that $E[\varepsilon_t|\varepsilon_{t-j}] = 0$ and hence $\text{Cov}(\varepsilon_t, \varepsilon_{t-j}) = 0$ for $j > 0$. We furthermore rule out collinearity between the cartel dummy

[7] See Perron (1989).

[8] Distribution theory is markedly different in the case of cointegration between price and its determinants, which we here exclude. Unreported findings, both analytical and by Monte Carlo simulation, suggest that similar conclusions apply also for nonstationary versions of DGP (1).

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D_t and x_t, which is a common assumption in the cartel literature. The main complication from having collinearity between elements in x_t and D_t is in the construction of the but-for price for p_t, which will then also depend on but-for price paths of those explanatory variables affected by the cartel. Note that by imposing orthogonality between cartel dummy and error term, Assumption 1 rules out the additional simultaneity bias that occurs when the formation of a cartel is jointly determined with prices and quantities. Alternatively, one can view model (1) as a reduced form price equation. Note that the presence of the cartel may also have changed the variability of prices. We therefore allow the conditional error variances σ_C^2 and σ_N^2 to be different between cartel and non-cartel regimes.

Once the coefficients of model (1) have been consistently estimated, but-for prices per period are predicted by their own lagged values, plus current empirical values of the additional covariates

$$\widehat{bfp}_t = \hat{\alpha}_1 + \hat{\beta}' x_t + \hat{\gamma} \widehat{bfp}_{t-1}, \quad t \in \mathbb{T}_C. \tag{8}$$

In the recursive dynamic simulation, the natural initial but-for price is the product unit price in the last period of competition before the start of the cartel.

Obviously, the estimated and actual but-for prices generally vary over time. We obtain that the but-for price estimator is consistent, however.

Proposition 1 *Under Assumption 1,*

$$\text{plim} \frac{1}{T_C} \sum_{t \in \mathbb{T}_C} \widehat{bfp}_t = \text{plim} \frac{1}{T_C} \sum_{t \in \mathbb{T}_C} bfp_t = E\left[bfp_t\right].$$

Given but-for prices, there are two different practical approaches to the estimation of overcharges in the literature. One is to compare but-for prices as in (8) to *predicted* cartel prices

$$\hat{p}_t = \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\beta}' x_t + \hat{\gamma} \hat{p}_{t-1}, \quad t \in \mathbb{T}_C, \tag{9}$$

constructed again using recursive dynamic simulation. They are predictions of the price in case of collusion ($D_t = 1$) and compared with those of the counterfactual competitive regime ($D_t = 0$).[9] The overcharge during collusion is then defined as

$$\widehat{O}_{1,t} = \hat{p}_t - \widehat{bfp}_t, \quad t \in \mathbb{T}_C. \tag{10}$$

The other approach is to compare estimated but-for prices as in (8) to *observed* prices p_t.[10] This defines the overcharge as

$$\widehat{O}_{2,t} = p_t - \widehat{bfp}_t, \quad t \in \mathbb{T}_C. \tag{11}$$

[9] This approach to calculating overcharges is taken, for example, in Nieberding (2006). For static models it reduces to the approach of David and Garces (2010), McCrary and Rubinfeld (2014) and Laitenberger and Smuda (2015).

[10] This approach to calculating overcharges is taken, for example, in Finkelstein and Levenbach (1983) and Harrington (2004).

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A crucial difference between the two overcharge types for further damage assessment is that $\widehat{O}_1$ is based on the constant cartel price effect α_2—as is the actual overcharge—in DGP (1), whereas $\widehat{O}_2$ implicitly includes the OLS residual and is period-specific.[11] Note that (10) and (11) together imply that $\widehat{O}_{2,t} = \widehat{O}_{1,t} + \hat{\varepsilon}_t$.

For correctly specified models, for which Assumption 1 holds and OLS estimators are consistent, there is no difference in expectation for the estimated overcharges. Under consistent estimation of the model parameters we have that

$$\operatorname{plim} \frac{1}{T_C} \sum_{t \in \mathbb{T}_C} \hat{\varepsilon}_t = 0. \tag{12}$$

Hence, average estimated overcharges defined as

$$\overline{O}_j = \frac{1}{T_C} \sum_{t \in \mathbb{T}_C} \widehat{O}_{jt}, \quad j = 1, 2, \tag{13}$$

converge to the average actual overcharge as defined in (7), as stated in the following proposition.

Proposition 2 *Under Assumption 1,*

$$\operatorname{plim} \overline{O}_1 = \operatorname{plim} \overline{O}_2 = \operatorname{plim} \overline{O}.$$

For either of the two different approaches to estimating overcharges, the total amount overcharged (5) can be estimated by

$$\widehat{CD}_j = \sum_{t \in \mathbb{T}_C} \widehat{O}_{jt} Q_t, \quad j = 1, 2. \tag{14}$$

In order to analyze the accuracy of these estimators, the relation between quantities and prices needs to be made explicit. Let the per period quantity purchased, Q_t, $t = 1, \ldots, T$, be price-dependent and stochastic with

$$\begin{aligned} E[Q_t] &= Q_C, \quad t \in \mathbb{T}_C, \\ E[Q_t] &= Q_N, \quad t \in \mathbb{T}_N. \end{aligned} \tag{15}$$

We then make the following assumption.

[11] It is not clear which overcharge type is advocated in the European Commission's 2013 *Practical Guide*. While in various places overcharges are defined, as in $\widehat{O}_2$, in comparison to the "price actually paid" (recital 79) and "the observed prices" (recital 101), the *Practical Guide* appears also to interpret the before-during-and-after method as generating a constant overcharge, which is more in line with overcharge definition $\widehat{O}_1$—see, for example, footnote 77.

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Assumption 2 *Quantities Q_t satisfy:*

(i) $Q_N \geq Q_C$*; and*

(ii) $E\left[Q_t \varepsilon_{t-j}\right] \leq 0, \quad t = 1, \ldots, T,\ j \geq 0.$

Assumption 2 (i) is the natural condition that in periods in which the cartel raised prices, the average quantity purchased will be less than or equal to the volume purchased under the lower competitive prices, i.e., that the demand curve is (weakly) downward sloping. Assumption 2 (ii) is a high-level assumption. It asks that Q_t is either negatively correlated or uncorrelated with current and past price shocks $\varepsilon_{t-j}, j \geq 0$. This is consistent with a (weak) negative correlation between price and quantity. Assumption 2 (ii) is satisfied under specific (sufficient) conditions that depend on the structure of demand. It holds (with equality), for example, when the demand curve is linear and (weakly) downward sloping, demand shocks are uncorrelated with prices and x_t is strictly exogenous. Assumption 2 (ii) can also hold in more complicated oligopoly settings where demand shocks are correlated with prices. For example, it is satisfied in the symmetric Cournot oligopoly model if the variance of cost shocks is larger than the variance of demand shocks.[12] For a general (weakly) downward sloping demand curve, Assumption 2 (ii) requires stronger independence conditions.

We are now ready for the first result on the damages estimators.

Theorem 1 *Under Assumptions 1 and 2,*

$$\text{plim}\,\frac{1}{T}\widehat{CD}_1 = \text{plim}\,\frac{1}{T}CD,$$

and

$$\text{plim}\,\frac{1}{T}\widehat{CD}_2 \leq \text{plim}\,\frac{1}{T}CD,$$

with strict equality if and only if $E\left[Q_t \varepsilon_{t-j}\right] = 0.$

The use of $\widehat{O}_{1t}$ as overcharge estimator returns a consistent damage estimator, irrespective of the structure of demand, as overcharge definition (10) is independent of quantities in all its components. Overcharge estimator $\widehat{O}_{2t}$ on the other hand does depend on Q_t, and therefore on the period-to-period interaction between price and quantity, through the OLS estimates of the error term. Only when price and quantity are uncorrelated is $\widehat{CD}_2$ a consistent estimator of CD. For regular demand, their correlation will be negative, implying that $\widehat{CD}_2$ underestimates true total overcharges. The overcharge method based on actual cartel prices therefore is conservative.

[12]See online Appendix A, Assumption 2(ii) under Linear Demand. We note that Assumption 2(ii) excludes intertemporal demand substitution.

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3 Misdating Bias

In this section we analyze the consequences of using a classification of cartel begin and end dates that is different from the actual dates that define the periods $\mathbb{T}_C$ and $\mathbb{T}_N$. When effective cartel and non-cartel periods are misclassified, the difference between overcharge calculation based on predicted and actual cartel prices turns out to matter fundamentally for cartel damage estimates as limit result (12) no longer holds.

In Section 3.1 we first consider the DGP (1) without autoregressive dynamics. We find that the bias of the overcharge estimator $\widehat{O}_1$ can have either sign, so that over- or underestimation of cartel damages can occur. The estimator $\widehat{O}_2$, however, robustly remains a conservative basis for calculating antitrust damages when the effective cartel periods are misclassified. Section 3.2 provides, in the simplest case of a comparison of average prices, the main intuition for these results. In Section 3.3 we extend the results of Section 3.1, for the case of a single cartel overcharge period, to the full model specification (1) including autoregressive dynamics.

3.1 Static cartel overcharges

Consider the DGP

$$p_t = \alpha_1 + \alpha_2 D_t + \beta x_t + \varepsilon_t, \quad t = 1, \ldots, T, \tag{16}$$

in which the dummy variable D_t is defined as in (2). Without loss of generality, we consider the case of a single explanatory variable x_t only. The cartel overcharge is a fixed positive margin α_2 over the competitive price level. We continue to assume that the regression model (16) is correctly specified, i.e. as in Assumption 1.

Suppose now that the cartel periods used in the estimation are misclassified in the sense that they are different from the effective cartel dates—we will refer to these dates as the formal cartel dates. That is, the estimated model for p_t is

$$p_t = \alpha_1 + \alpha_2 d_t + \beta x_t + u_t, \quad t = 1, \ldots, T. \tag{17}$$

Depending on the values for D_t and d_t any number of different misdating scenarios is possible: around each period with cartel overcharges and across, the formal begin and end dates may be either both earlier, both later, or respectively earlier and later, or vice versa, than the effective dates. Define

$$\lambda_{ij} = \frac{1}{T} \sum_{t=1}^{T} I\left(d_t = i, D_t = j\right), \qquad i = 0, 1; j = 0, 1, \tag{18}$$

with $I(\cdot)$ the indicator function. That is, λ_{ij} is the fraction of observations for which scenario ij occurs. We furthermore define $\mathbb{T}_{ij}$ as the collection of observations for scenario

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ij, which has T_{ij} observations. For the effective cartel and non-cartel periods we have $\mathbb{T}_C = \mathbb{T}_{01} \cup \mathbb{T}_{11}$ and $\mathbb{T}_N = \mathbb{T}_{00} \cup \mathbb{T}_{10}$. Also we define $\mathbb{T}_c = \mathbb{T}_{10} \cup \mathbb{T}_{11}$ and $\mathbb{T}_n = \mathbb{T}_{00} \cup \mathbb{T}_{01}$ to indicate the formal cartel and non-cartel periods with $T_c = T_{10} + T_{11}$ and $T_n = T_{00} + T_{01}$ observations respectively.

Obviously, if $\lambda_{01} = 0$ and $\lambda_{10} = 0$, the OLS estimators are unbiased and consistent. To analyze the effects of using formal cartel dates that are different from effective cartel dates, i.e. $\lambda_{01} > 0$ and/or $\lambda_{10} > 0$, we adapt the approach from Perron (1989) on neglected structural breaks to the more general framework of mismeasured structural change. We obtain the following result on the OLS coefficient estimators of model (17) when the DGP is (16).

Lemma 1 *Let $\lambda_{01} > 0$ and/or $\lambda_{10} > 0$. Under Assumption 1:*

$$\begin{aligned}
\text{plim}\, \hat{\alpha}_1 &= \alpha_1, \quad \lambda_{01} = 0, \\
\text{plim}\, \hat{\alpha}_1 &> \alpha_1, \quad \textit{otherwise}, \\
\text{plim}\, \hat{\alpha}_2 &< \alpha_2, \quad \forall \lambda_{01}, \lambda_{10}, \\
\text{plim}\, \hat{\beta} &= \beta, \quad \forall \lambda_{01}, \lambda_{10}.
\end{aligned}$$

Misdating the cartel dummy regressor d_t in equation (17) leads to an attenuation bias: the estimator of the dummy coefficient α_2 is biased downward, irrespective of the direction of the misdating of the effective cartel periods. In addition, misdating implies that the estimator of the competitive price level α_1 is biased upwards, except when the formal cartel periods encompass the effective periods ($\lambda_{01} = 0$), in which case the estimator is consistent. The misdating bias in α_2 does not carry over to the estimation of β, however.

Lemma 1 implies that estimators of but-for prices and overcharges are biased too, as stated in the following proposition.

Proposition 3 *Under the conditions of Lemma 1:*

(i)

$$\begin{aligned}
\text{plim}\, \frac{1}{T_c} \sum_{t \in \mathbb{T}_c} \widehat{bfp}_t &= E\left[bfp_t\right], \quad \lambda_{01} = 0, \\
\text{plim}\, \frac{1}{T_c} \sum_{t \in \mathbb{T}_c} \widehat{bfp}_t &> E\left[bfp_t\right], \quad \textit{otherwise.}
\end{aligned}$$

(ii)

$$\begin{aligned}
\text{plim}\, \overline{O}_1 &< \overline{O}, \\
\text{plim}\, \overline{O}_2 &< \overline{O}.
\end{aligned}$$

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Using cartel dates in the before-during-and-after method that are different from the effective cartel dates results in estimated but-for prices that are on average higher than the actual but-for price, except when the formal cartel periods encompass the effective cartel periods, in which case they are correct on average. Note that while on average but-for prices are higher, in individual time periods but-for prices can be lower using formal dates. In addition, Proposition 3 shows that the average overcharge is strictly underestimated for all types of misdating. The inequalities are strict, including the case $\lambda_{01} = 0$, as the average is taken over the formal cartel periods, and so always includes misclassified data.

The effects that misclassifying the effective cartel period(s) has on total damages is twofold, since the total overcharge is determined by the period overcharge(s), as well as by the number of periods designated as collusive, with a quantity purchased each. The overcharge definition used, $\widehat{O}_{1t}$ or $\widehat{O}_{2t}$, turns out to matter materially for the consequences of misdating, as shown in the following result.

Theorem 2 *Under the conditions of Lemma 1 and Assumption 2:*

(i)

$$\begin{aligned} \text{plim}\,\frac{1}{T}\widehat{CD}_1 &\geq \text{plim}\,\frac{1}{T}CD, \quad \lambda_{01} = 0, \\ \text{plim}\,\frac{1}{T}\widehat{CD}_1 &< \text{plim}\,\frac{1}{T}CD, \quad \lambda_{10} = 0, \\ \text{plim}\,\frac{1}{T}\widehat{CD}_1 &\gtreqless \text{plim}\,\frac{1}{T}CD, \quad \textit{otherwise}. \end{aligned}$$

(ii)

$$\begin{aligned} \text{plim}\,\frac{1}{T}\widehat{CD}_2 &\leq \text{plim}\,\frac{1}{T}CD, \quad \lambda_{01} = 0, \\ \text{plim}\,\frac{1}{T}\widehat{CD}_2 &< \text{plim}\,\frac{1}{T}CD, \quad \textit{otherwise}. \end{aligned}$$

The estimator $\widehat{CD}_1$ can under- as well as overestimate the actual damage, because the overcharge $\widehat{O}_{1t}$ is applied to a sum total of quantities labelled as purchased under the cartel regime that may be higher or lower than the actual total quantity. The estimator $\widehat{CD}_2$, on the other hand, for all misdating scenarios provides a conservative estimate, in the sense that the actual total damage during the effective cartel period(s) is at least as large as the damage estimated using misclassified dates in the but-for price estimation. The reason for this difference between $\widehat{CD}_1$ and $\widehat{CD}_2$ is that $\widehat{O}_{2t}$ is the sum of $\widehat{O}_{1t}$ and the OLS residual, which now picks up the measurement error. In addition to (weakly) underestimating the actual overcharge, because the overcharge $\widehat{O}_{2t}$ is period-specific, in misdated estimations that lead to higher than actual but-for prices (i.e. in all cases but $\lambda_{01} = 0$) this overcharge

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becomes negative in competitive periods falsely labelled as collusive (i.e. for $t \in \mathbb{T}_N$, where $\alpha_2 = 0$), thus decreasing the total damage estimate. As in Theorem 1, the estimator $\widehat{CD}_2$ is consistent for CD only when $E[Q_t \varepsilon_t] = 0$.

When the dates are correct, by Theorem 1, $\widehat{CD}_1$ is a consistent estimator. Hence all bias in $\widehat{CD}_1$ established in Theorem 2 is due to misdating. For $\widehat{CD}_2$, Theorem 1 showed a downward bias already when price and quantity are negatively correlated. The misdating bias in $\widehat{CD}_2$ is therefore the sum of this bias under correct dating and additional bias due to misdating. Only when $\lambda_{01} = 0$ is the latter effect zero. In general the relative size of the two sources of bias depends on the own-price elasticity of demand and the extent of the misdating.

3.2 Illustration by comparison of mean prices

To gain intuition for the source of the misdating biases, consider the special case without covariates

$$p_t = \alpha_1 + \alpha_2 d_t + u_t. \tag{19}$$

Let $\bar{p}_c$ be the mean price over the formal cartel periods and $\bar{p}_n$ be the mean price over the formal non-cartel periods. The OLS estimators of model (19) can be expressed as

$$\hat{\alpha}_1 = \bar{p}_n, \quad \hat{\alpha}_2 = \bar{p}_c - \bar{p}_n. \tag{20}$$

Misspecifying the effective cartel dates amounts to falsely labelling part of the competitive prices as collusive and/or part of the collusive prices as competitive. So including higher than competitive prices in $\bar{p}_n$ results in an upward bias in estimating α_1, the competitive price level, because $\bar{p}_n$ overestimates the true mean price in the non-cartel periods. Only when $\lambda_{01} = 0$ is $\hat{\alpha}_1$ based on non-cartel data only, so that it is unbiased.

For the same reason, mistakenly including competitive prices in the cartel sample decreases the average $\bar{p}_c$. This is the case in all misdating scenarios but $\lambda_{10} = 0$, when the effective cartel periods include the formal cartel periods. The estimated cartel effect $\hat{\alpha}_2$, which is the difference in average price between cartel and non-cartel periods, has a downward bias in all misdating cases, as either $\bar{p}_c$ or $\bar{p}_n$ is affected, or both. When $\bar{p}_c$ is correct ($\lambda_{10} = 0$), $\bar{p}_n$ is higher than the true mean non-cartel price, and when $\bar{p}_n$ is correct, $\bar{p}_c$ is lower than the true mean cartel price ($\lambda_{01} = 0$).

Figure 2 illustrates for a single cartel period how the use of estimator $\widehat{O}_{2t}$ as a basis for damage calculation is conservative. The actual cartel price increase by α_2 lasts from T_B to T_E. The vertical dashed lines at T_b and T_e are the formal cartel dates used in the estimation. We consider the misdating case in which both the begin and end of the cartel

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are formally dated too early ($\lambda_{01} > 0$ and $\lambda_{10} > 0$), but the effective and formal cartel periods still overlap.

The average actual but-for price is α_1, yet by using the misclassified formal cartel dates, the estimator $\hat{\alpha}_1$ uses price data from prior to T_b and past T_e. The estimate therefore includes observations between T_e and T_E that are actually cartel prices, but falsely labelled competitive prices. Inclusion of these higher prices increases the estimated but-for price $\widehat{bfp}_t$ above the level of the actual but-for price.

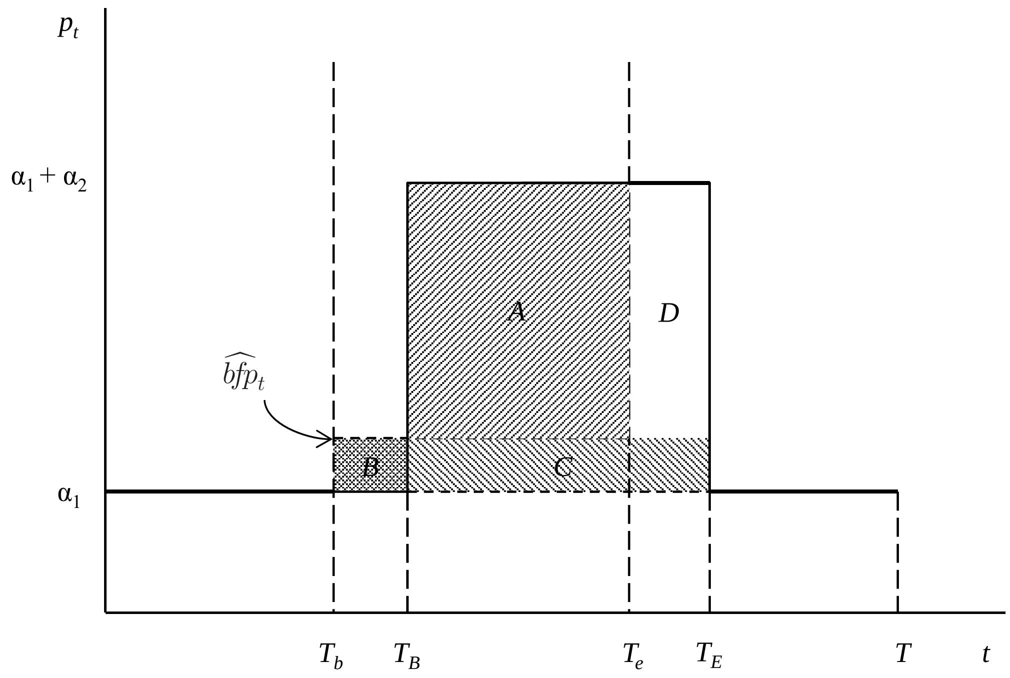

Figure 2: Theorem 2 illustrated

For three reasons, therefore, the estimated damage total is smaller than the actual damage. First, the overcharges are smaller by area C, because the but-for price is overestimated. Second, the formal cartel end date T_e lies before the effective cartel end date T_E, so that part D is missed. Third, during the period $T_b < T_B$, the overestimation of the but-for price results in negative overcharges the size of area B, which is subtracted from the damage estimate area A.

The damage total $\widehat{CD}_2$ underestimates CD even when the formal cartel period is longer than the effective. This can be illustrated in Figure 2 by moving T_b further to the left of T_B, other things equal. It enlarges the area B subtracted. The effect of this larger misdating error on $\widehat{CD}_1$, however, is ambiguous. While $\widehat{O}_{1t}$ is lower, the earlier formal cartel begin date brings more of the larger competitive volume under the damage period. These effects

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trade off, depending among other things on the own-price elasticity of demand.

When $\lambda_{10} = 0$, in which the formal cartel period falls entirely within the effective period, there are no negative overcharges, yet the but-for price is grossly overestimated and two periods of actual damages left and right from the effective cartel dates are not counted. Only when the formal period includes the effective period, i.e. $\lambda_{01} = 0$, will the $\widehat{O}_{2t}$ overcharges calculated outside the effective cartel period be zero on average, thereby not contributing to the damage estimate, so that $\widehat{CD}_2$ is an unbiased estimator of actual damages. $\widehat{CD}_1$, however, may still overestimate the actual damage in this case, as $\bar{p}_c$ (and so $\widehat{O}_{1t}$) decreases in the length of the formal cartel period, while at the same time more purchases are alleged to be affected.

3.3 Full price dynamics with one overcharge

Now consider the full model specification (1), including the lagged value of price. First note that there is no reason to think that estimator $\widehat{CD}_1$ will improve in the extended model, and so we need not examine it further. To see if $\widehat{CD}_2$ remains a robust conservative estimator, consider the estimated model

$$p_t = \alpha_1 + \alpha_2 d_t + \beta x_t + \gamma p_{t-1} + u_t, \quad t = 1, \ldots, T. \tag{21}$$

Autoregressive dynamics in the but-for price series implies that allowing for multiple cartel periods ($R > 1$) would lead to a substantial increase in the types of scenarios to be considered, as misdating one cartel period potentially has ambiguous lasting effects on later episodes. To avoid this complexity, and in keeping with our case study in Section 5, we limit the analysis to a single period of cartel effects as defined in (2).[13] Using misclassified cartel dates in the estimation (21) with $R = 1$ means that the cartel dummy variable d_t takes on the following values:

$$d_t = \begin{cases} 0, & t \leq T_b, \\ 1, & T_b < t \leq T_e, \\ 0, & t > T_e, \end{cases} \tag{22}$$

where T_b is the formal cartel begin date and T_e the formal cartel end date.

Obviously, if $\lambda_{01} = \lambda_{10} = 0$ ($T_b = T_B$ and $T_e = T_E$) the OLS estimators are consistent. There are four possible misdating scenarios for the break dates in total. We obtain the following results on the OLS coefficient estimators of model (21) when the DGP is (1).

[13]Monte Carlo experiments for dynamic models with $R > 1$ suggest that the conclusions of Lemma 2, Proposition 4 and Theorem 3 continue to hold.

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Lemma 2 *Let $\lambda_{01} > 0$ and/or $\lambda_{10} > 0$ and $R = 1$. Under Assumption 1:*

$$\begin{aligned}
&\text{plim}\, \hat{\alpha}_1 < \alpha_1, \\
&\text{plim}\, \hat{\alpha}_2 < \alpha_2, \\
&\text{plim}\, \hat{\beta} \lesseqgtr \beta, \quad \textit{if } m_{xp_{-1}} \gtreqless 0, \\
&\text{plim}\, \hat{\gamma} > \gamma,
\end{aligned}$$

where $m_{xp_{-1}} = \text{plim}\, \frac{1}{T}\sum_{t=1}^{T} x_t p_{t-1}$.

Misdating bias now occurs in all coefficients. The cartel coefficient α_2 continues to be underestimated in all misdating scenarios. The intercept α_1 now is always underestimated as well. The autoregressive coefficient γ is overestimated always, whereas the direction of the bias in the coefficient β of the explanatory variable depends on the sign of the particular data moment $m_{xp_{-1}}$.

With bias in the OLS estimators of all regression coefficients, it is not obvious what the impact is of misspecified cartel begin and end dates on the estimated but-for prices, cartel overcharges and total damage. We obtain the following for the average but-for price and average overcharge.

Proposition 4 *Under the conditions of Lemma 2:*

(i)

$$\begin{aligned}
&\text{plim}\, \frac{1}{T_c}\sum_{t\in\mathbb{T}_c} \widehat{bfp}_t = \text{plim}\, \frac{1}{T_c}\sum_{t\in\mathbb{T}_c} bfp_t, \quad \lambda_{01} = 0, \\
&\text{plim}\, \frac{1}{T_c}\sum_{t\in\mathbb{T}_c} \widehat{bfp}_t > \text{plim}\, \frac{1}{T_c}\sum_{t\in\mathbb{T}_c} bfp_t, \quad \textit{otherwise.}
\end{aligned}$$

(ii)

$$\text{plim}\, \overline{O}_2 < \text{plim}\, \overline{O}.$$

Hence, but-for prices continue to be overestimated in all misdating scenarios but with inclusive formal dates, and the average overcharge is underestimated in all cases. Moreover, for the preferred overcharge estimator $\widehat{O}_{2t}$ we find the following analogue to Theorem 2.

Theorem 3 *Under the conditions of Lemma 2 and Assumption 2:*

$$\begin{aligned}
&\text{plim}\, \frac{1}{T}\widehat{CD}_2 \leq \text{plim}\, \frac{1}{T} CD, \quad \lambda_{01} = 0, \\
&\text{plim}\, \frac{1}{T}\widehat{CD}_2 < \text{plim}\, \frac{1}{T} CD, \quad \textit{otherwise.}
\end{aligned}$$

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We conclude that when the effective cartel dates are unknown, the preferred approach to cartel damages estimation is to compare estimated but-for prices to *observed* prices, not *predicted* cartel prices. The estimator $\widehat{CD}_2$ applied to the misclassified cartel dates provides a lower bound for the actual cartel damages, irrespective of the type and size of the misdating. $\widehat{CD}_2$ furthermore is consistent as long as the formal cartel dates used in the estimation encompass the effective cartel dates. In conclusion, when the effective cartel dates are unknown, the preferred approach to cartel damages estimation is to use conservative overcharge estimator $\widehat{O}_{2t}$.

4 Inference with Structural Breaks of Unknown Number and Dates

The bias from misspecifying effective cartel begin and end dates can be mitigated by determining the effective dates with more precision. One way to do this is to supplement the before-during-and-after method with tests for multiple structural change. Conditional on the estimated number of breaks and estimated break dates, an alternative cartel dummy regressor is constructed, which is then used in the empirical analysis to estimate cartel effects. In this section we discuss the accuracy of this empirical strategy in more detail.

We use Bai and Perron (1998, 2003) test procedures for detecting *a priori* unknown multiple structural breaks. Bai and Perron (1998, 2003) suggest to model any serial correlation parametrically by including lagged regressors.[14] While Bai-Perron tests can in principle also be used to determine possible changes in (some or all) coefficients of the explanatory variables, in line with the cartel literature we treat the difference between cartel and non-cartel regimes as price level shifts.

Consider a general version of DGP (1) with up to $m+1$ different intercepts

$$p_t = \delta_j + \beta' x_t + \gamma p_{t-1} + \varepsilon_t, \qquad t = T_{j-1}+1, \ldots, T_j, \tag{23}$$

for $j = 1, \ldots, m+1$ with $T_0 = 0$ and $T_{m+1} = T$. As the intercepts equal $\delta_1, \ldots, \delta_{m+1}$, the model allows for $m+1$ regimes with unknown break dates $T_1, \ldots, T_m$.[15]

Given a set of candidate break dates $T_1, \ldots, T_m$, the least-squares estimates minimize the residual sum of squares from (23)

$$S_T(T_1, \ldots, T_m) = \sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} \left(p_t - \delta_j - \beta' x_t - \gamma p_{t-1}\right)^2. \tag{24}$$

[14] As an alternative approach to treating potential serial correlation in the time series, Bai and Perron (1998, 2003) suggest to estimate a static model and treat serial correlation non-parametrically by exploiting HAC covariance matrix estimation.

[15] Note that in model (1) $m = 2$, implying $\delta_1 = \alpha_1$, $\delta_2 = \alpha_1 + \alpha_2$ and $\delta_3 = \alpha_1$.

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The effective cartel dates $\left(\hat{T}_1, \ldots, \hat{T}_m\right)$ are found as

$$\left(\hat{T}_1, \ldots, \hat{T}_m\right) = \text{argmin}_{T_1,\ldots,T_m} S_T\left(T_1, \ldots, T_m\right), \tag{25}$$

where the minimization is over all partitions for which $T_j - T_{j-1} \geq h$, the trimming parameter h being the minimal number of observations between two breaks.

The null hypothesis of no break is tested against the alternative hypothesis of m breaks, using the standard F-test statistic for goodness of fit of the model with and without the estimated break dates $\left(\hat{T}_1, \ldots, \hat{T}_m\right)$, developed in Chow (1960) for known break dates. This is referred to as the sup F-test: the maximum over all possible Chow F-tests. It has a non-standard asymptotic distribution, for which asymptotic critical values are provided in Bai and Perron (1998). Perron (2006) conjectures that most tests for structural change will have non-monotonic power when the true number of breaks is larger than the number of breaks imposed in the construction of the test statistic. It is advisable, therefore, to report outcomes of the sup F-test for a number of different choices of m. As the number of breaks in the data is not *a priori* known, use is made of the unweighted and weighted double maximum tests, labeled UD max and WD max, which are the maxima of a series of sup F-statistics, to test the null hypothesis of no break versus an unknown number of breaks—up to some prespecified maximum.

The break dates so found indicate structural changes in the specification, such as would be caused by a regime switch from competition to collusion, and *vice versa*. The procedure thus allows for establishing empirically whether there were effects of the cartel violation and whether they were continuous, or rather consisted of several collusive episodes with lower price intermittants. It is important to note that in the dynamic specification, the breaks in the time series should be interpreted as the dates where the cartel began to become effective, or started to lose its effectiveness. The transitions from competitive to collusive prices and back typically are gradual. For various reasons discussed, price levels may continue to be affected by a cartel long dissolved. The end date of such lasting effects of a cartel would be when the but-for prices and the actual prices have fully converged.[16]

A concern about the test procedures is that they rely on asymptotic distribution theory. Monte Carlo simulation studies on univariate time series models have shown that finite sample distributions of the various test statistics may deviate substantially from their asymptotic approximations.[17] In particular, size distortions are found to grow (and power to decrease) when time series become more persistent.[18] In principle, the bootstrap can

[16] In a landmark decision in a paper case in 2010, the German Federal Court of Justice allowed to take "lingering effects" of the cartel into account as a factor augmenting the damage—see German Federal Court of Justice, 28 June 2011, ORWI, recital 84. See also Hüschelrath et al. (2013).

[17] See Diebold and Chen (1996), O'Reilly and Whelan (2005), and Bai and Perron (2006).

[18] Additionally, Bai and Perron (2006) report Monte Carlo simulation results showing that the use of

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be used to get a better approximation of the finite sample distribution of the structural break tests. Diebold and Chen (1996) show that for the univariate first-order autoregressive model, using a standard nonparametric bootstrap leads to tests with the correct size.

To the best of our knowledge, no such simulation results exist for asymptotic and bootstrap Bai-Perron tests for multiple regression models with autoregressive short-run dynamics and additional explanatory variables. We have therefore analyzed in a small scale Monte Carlo study for our DGP (1) whether the implementation of structural break tests for detecting and dating cartel effects has enough size control. The results are reported in online Appendix B.

The actual size of the statistical tests for structural change is found to be close to nominal significance levels when the data are moderately persistent. Size distortions occur, however, when there is high persistence, which could lead to spurious breaks. Yet the bootstrap can effectively deal with this problem and leads to test procedures with the correct size. The simulations also show that the bootstrap tests have nontrivial power, at least for the cases examined. Therefore, in practice the bootstrap version of the break tests should be favored.

We have also analyzed by simulation the accuracy of OLS inference based on the estimated break dates. We focus on the bias of the OLS coefficient estimator of the cartel effect α_2, and the size distortion of the corresponding t-ratio. Finite sample bias and size distortions are small in case of moderately persistent data. Even in case of high persistence only moderate coefficient bias and size distortions occur, hence the accuracy of OLS inference based on estimated break dates is satisfactory.

5 Dating the *Sodium Chlorate* Cartel

In June 2008, the European Commission adopted a cartel infringement decision in *Sodium Chlorate* against four groups of chemical producers, imposing fines of 79 million Euro in total.[19] Sodium chlorate is a chemical compound synthesized from the electrolysis of salt and water. It is applied in the bleaching of pulp to make it suitable for the manufacture of printing and tissue grade papers that meet the Elemental Chlorine Free (ECF) paper standard. The cost of sodium chlorate as part of the final chemical pulp prices is about 1%. Supply agreements typically took the form of a medium-term framework contract (usually 3–5 years), including estimated purchase volumes and a clause for price adjustments based

nonparametric covariance matrix estimators leads to substantially larger size distortions than modeling serial correlation parametrically by using dynamic regression models.

[19]European Commission decision of 11/06/2008 in Case COMP/38.695 – *Sodium Chlorate*. The decision was upheld by the European General Court.

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on specified cost indicators. Price revisions were made throughout the year and common for the new calendar year.

When demand began to stagnate in anticipation of the stricter Totally Chlorine Free (TCF) paper standard, and (expected) overcapacity put downward pressure on prices in the first half of the 1990s, the main sodium chlorate producers in Europe formed a cartel to implement a strategy of stabilizing the market. The first cartel meeting of which the Commission obtained evidence was on 21–22 September 1994, in Helsinki, in which Akzo/EKA and Kemira agreed on upward price adjustments for 1995. The cartel expanded in 1995 and 1996, when smaller suppliers joined. Jointly, the cartel members served over 90% of the total sodium chlorate market in Europe.

In 1998 some internal tensions are reported to have risen between the cartel members, after one sodium chlorate producer was suspected to have secretly supplied another's customer. Repeated attempts were made to maintain cartel stability, leading to coordinated price increases in 1997, 1998 and 1999. In March 2003, Akzo/EKA brought the existence of the cartel to the attention of the European Commission with a successful leniency application. The Commission determined in its decision that the infringement had formally ended February 9th, 2000, when Akzo/EKA had denounced the cartel at a trade association meeting.

The monthly price per ton of sodium chlorate is analyzed in the sample period January 1993 to December 2005 on the basis of $T = 156$ volume-weighted average delivered price observations (p_t). The price data were obtained from several large direct purchasers of the cartel, which together exercised almost half of total European demand for sodium chlorate in the relevant period. Monthly variation in the price series reflects (contractual) price revisions made throughout the year. The price data do not exhibit a seasonal pattern.

We estimate the dynamic regression model

$$p_t = \alpha_1 + \alpha_2 d_t + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \gamma_1 p_{t-1} + \gamma_2 p_{t-2} + u_t, \qquad (26)$$

in which d is the cartel dummy, x_1 is the electricity price, x_2 labour costs, x_3 Western European chemical pulp production, and x_4 European production capacity for sodium chlorate.[20] Electricity and labour are the major input costs in the production of sodium chlorate and the main common contract indexation factors, while pulp production and sodium chlorate production capacity serve as demand and supply shifters, respectively. The short-run dynamics of product prices are modelled by two lagged price regressors p_{t-1} and p_{t-2}. Except for the cartel dummy, all variables are measured in logarithms, so that the regression coefficients are (semi-)elasticities.

[20]Sources are respectively: Eurostat, "Electricity - industrial consumers - half-yearly prices"; OECD System of Unit Labour Cost Indicators; CEPI, European Chemical Pulp Production, 1995Q1 - 2008Q3; Harriman Chemsult, Chemicals Economics Handbook–SRI Consulting and CMAI.

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The cartel effect is best modelled with just an intercept shifter. Allowing, conditional on the found dates, for breaks in the parameters of the control variables or lagged price regressors returns no significant differences except for the coefficient of labour costs, for which results for average but-for price and overcharge change only marginally, implying that its economic significance is negligible.[21] We therefore maintain the assumption of constant slope coefficients in the construction of but-for prices and overcharges.

Table 1 reports the results of various structural break tests on the intercept. To avoid possible low power due to an underspecified number of breaks, we report sup F-test outcomes for $m \in \{1, 2, 3, 4, 5\}$. We use two different trimming parameters $\mu = h/T \in \{0.10, 0, 15\}$. Furthermore, the UD max statistic is used to determine the number of breaks. For both asymptotic and bootstrap critical values, all sup F-tests as well as the UD max test indicate two, and only two, break points, estimated at January 1995 and February 2002, marking structural changes in the effectiveness of the cartel.

Table 1: Empirical outcomes from structural break tests in *Sodium Chlorate*

	homogeneity		heterogeneity		asymptotic cv		bootstrap cv	
μ	0.10	0.15	0.10	0.15	0.10	0.15	0.10	0.15
sup $F(1)$	1.36	1.36	1.29	1.29	9.10	8.58	15.27	14.75
sup $F(2)$	27.34	8.44	29.71	10.07	7.92	7.22	12.67	11.81
sup $F(3)$	22.81	3.54	24.24	3.87	6.84	5.96	11.96	10.15
sup $F(4)$	21.01	4.90	21.82	5.49	6.03	4.99	11.45	8.97
sup $F(5)$	20.16	4.04	19.93	4.10	5.37	3.91	10.94	6.81
UD max	27.34	8.44	29.71	10.07	9.52	8.88	16.01	15.35
#breaks	2	0	2	2				
break dates	1995:1		1995:1	1995:1				
	2002:2		2002:2	2002:2				

Note: nominal size is 5%. Bootstrap critical values (cv) are calculated using 1000 replications.[22] $\mu = h/T$ is the trimming parameter.

Figure 3 displays the actual sodium chlorate prices as the solid line p. The effective cartel dates estimated are indicated with solid vertical lines, while the dashed vertical lines are the formal cartel begin and end dates—September 1994 and February 2000. But-for prices using the effective (formal) cartel dates are bfp (bfpo).

Sodium Chlorate is a single and continuous period of cartel effects of which both the begin and the end are dated too early, which is the case illustrated in Section 3.2. The effective cartel begin date matches the Commission's evidence that the initial cartel meetings

[21] See Godek (2011).

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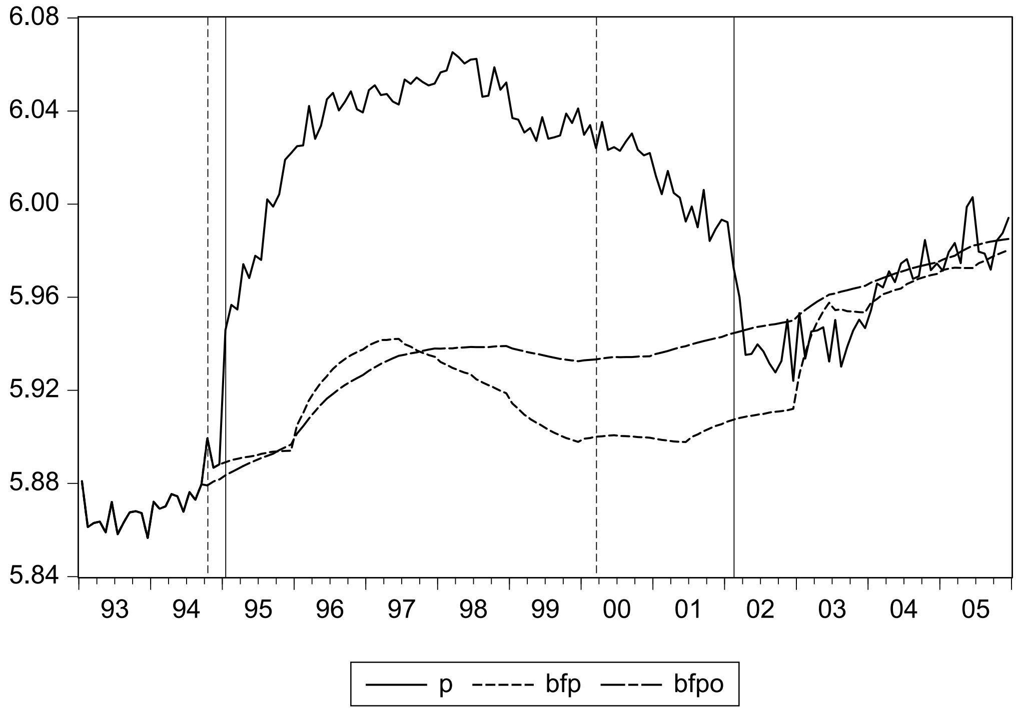

Figure 3: Cartel price effects in *Sodium Chlorate*

in September 1994 aimed to raise prices as of the following year: prices increase steeply from the beginning of the year 1995, to peak in 1998. Even though prices subsequently declined in the second half of 1998—in fact with electricity prices, the main cost component in the production of sodium chlorate—they stabilized with the reported resolve of internal stability issues. The effects of the cartel on prices subsequently persisted until the beginning of 2002, when sodium chlorate price fell rapidly, almost two years after the formal end date February 9th, 2000. By the time the existence of the cartel was notified to the European Commission, in March 2003, prices had returned to a more stable competitive trend.

We estimate the dynamic model (26) with OLS for two date specifications of the cartel dummy. Using the formal dates, i.e. September 1994 – February 2000, we obtain

$$\hat{p}_t = \underset{(0.358)}{0.783} + \underset{(0.003)}{0.013} d_t + \underset{(0.011)}{0.007} x_{1t} + \underset{(0.016)}{0.023} x_{2t} - \underset{(0.019)}{0.005} x_{3t} - \underset{(0.059)}{0.054} x_{4t} + \underset{(0.077)}{0.596} p_{t-1} + \underset{(0.074)}{0.299} p_{t-2}. \quad (27)$$

While using the effective cartel dates, i.e. January 1995 – February 2002, we find

$$\hat{p}_t = \underset{(0.369)}{2.033} + \underset{(0.004)}{0.029} D_t + \underset{(0.011)}{0.056} x_{1t} + \underset{(0.013)}{0.031} x_{2t} + \underset{(0.017)}{0.002} x_{3t} - \underset{(0.053)}{0.145} x_{4t} + \underset{(0.074)}{0.455} p_{t-1} + \underset{(0.067)}{0.300} p_{t-2}. \quad (28)$$

Using the formal cartel dates returns coefficients of the explanatory variables that are all insignificant, whereas the specification does contain the primary cost factors known to determine price in the sodium chlorate industry. Applying the effective cartel dates instead,

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coefficients all have the expected sign. Capacity is the most important determinant of price, followed by electricity costs.

The empirical model has strong combined explanatory power, with a large F statistic (827.70) on the overall significance of the regression. For both specifications, the estimated short-run dynamics are adequate, i.e. additional lags of the regressors are not significant. Furthermore, Breusch-Godfrey Lagrange Multiplier misspecification tests do not indicate significant residual autocorrelation, which indicates that all observed autocorrelation in the price has been modelled parametrically by the estimated dynamic specification. The predictive power of the estimated dynamic model is superior to the static specification. Various sensitivity checks indicate that the estimation results are robust to changes in functional form and sample period.

Comparing the estimates in (27) and (28), their relative magnitude corroborates the results of Lemma 2. The short-run cartel effect $\hat{\alpha}_2$ is more than twice as large using the effective cartel dates rather than the formal cartel dates (2.9% versus 1.3%). The sum of the autoregressive coefficients is also higher under the formal cartel date specification, while the estimated intercept $\hat{\alpha}_1$ is lower.

Using the effective cartel dates specification, but-for prices are constructed by recursive dynamic simulation as

$$\widehat{bfp}_t = 2.033 + 0.455\,\widehat{bfp}_{t-1} + 0.300\,\widehat{bfp}_{t-2} + 0.056\,x_{1t} + 0.031\,x_{2t} + 0.002\,x_{3t} - 0.145\,x_{4t},$$

from January 1995 onwards. The observed prices in November and December 1994 are used as initial values of the period-by-period but-for simulation, i.e. $\widehat{bfp}_{1994:11} = p_{1994:11}$ and $\widehat{bfp}_{1994:12} = p_{1994:12}$. In Figure 3, these are plotted as the dotted line (labelled bfp). While indeed the cartel started to lose its effectiveness in raising price in February 2002, the but-for and the actual price series are fully converged only from February 2003, which identifies the end date of the cartel effect. This lingering effect is consistent with contracts and negotiations common in the industry, as well as the late leniency application.

Similarly, but-for prices using the formal cartel dates are simulated as the dash-dotted line (labelled bfpo). On average, the but-for prices estimated on the basis of the formal dates are 0.95% higher than the but-for prices corresponding to the effective dates, which corroborates Proposition 3(i)—notwithstanding the fact that the formal dates but-for prices are lower than the effective dates but-for prices during the period 1996–1997.

The overcharge estimates are also in conformity with the theoretical predictions. Using the effective dates, we find that the average $\widehat{O}_1$ and $\widehat{O}_2$ overcharges are almost equal, corroborating Proposition 2. Moreover, using the formal cartel dates, the average $\widehat{O}_1$ and $\widehat{O}_2$ overcharges are respectively 9.33% and 8.02% lower than their counterparts using the effective dates, which is in line with Proposition 3(ii). The downward bias in overcharge

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estimation is more or less equal for $\widehat{O}_1$ and $\widehat{O}_2$.

Total damage estimates $\widehat{CD}_1$ and $\widehat{CD}_2$ are almost identical under the effective dates. From Theorem 1, this indicates that sodium chlorate demand has a very low own-price elasticity. Indeed, estimation of a dynamic regression of sodium chlorate quantities on prices shows an insignificant price effect. This is consistent with demand for sodium chlorate use in bleaching being driven by mainly by pulp and paper production and prices, the availability of only imperfect and more expensive substitutes, and the cost for sodium chlorate being a negligible cost factor in chemical pulp production.

The damage estimates $\widehat{CD}_1$ and $\widehat{CD}_2$ are respectively 28.32% and 28.57% lower using the formal rather than the effective cartel dates, which is consistent with Theorem 3. The largest part of this difference is due to the fact that the formal end date lies before the effective end date, i.e. area D in Figure 2, which accounts for 25.68% and 23.95% lower damage estimates $\widehat{CD}_1$ and $\widehat{CD}_2$. These significant differences clearly illustrate the anomalous effects of misspecifying cartel begin and end dates: it results in downward bias that can be large in comparison to proper dating. The bias is due to both lower overcharges and a shorter damage period.

6 Concluding Remarks

We have shown that proper cartel dating is crucial for obtaining accurate estimates of cartel damages. Using misclassified cartel begin and end dates leads to a (weak) overestimation of but-for prices and an underestimation of overcharges. When overcharges are defined as the difference between *actual* prices and but-for prices, the resulting damage estimator is conservative in the sense that it always leads to a (weak) underestimation of the true damage. While longer formal cartel periods subsume more volume under the damage claim, but-for prices are (weakly) overestimated, so that volumes purchased in falsely alleged cartel period parts are premultiplied by non-positive overcharges. Instead, using the difference between *predicted* cartel prices and but-for prices, both over- and underestimation of cartel damages can occur.

The misdating bias can be avoided with econometric tests for structural change. Even with proper empirical cartel dating though, when the effective cartel dates are unknown, the preferred approach to cartel damages estimation is to compare estimated but-for prices to observed prices. The approach is conservative when the effective cartel dates are exactly estimated, and remains so for an error margin. It is more robust against misdating than comparisons with predicted cartel prices, which recent reports appear to favor. The empirical findings in *Sodium Chlorate* corroborate our theoretical results. Using the formal cartel dates estimates damages more than 25% lower.

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Under tort law, the injured party is entitled to compensation from the wrongdoer for harm that is caused by—and typically follows—the tortious act. Damages resulting from the collusion outside the formal cartel period(s) should therefore in principle be permitted as part of an antitrust claim, including post-cartel tacit collusion and lingering effects. Together with qualitative evidence of the cartel's *modus operandi*, cartel dating can help corroborate or falsify the collusion as the legally recognizable cause of the harm. Plaintiffs in an antitrust damages action should apply dating also to avoid the risk of leaving part of their actual cartel damages unclaimed. Defendants can use the techniques to demonstrate an alleged cartel ineffective. Agencies may want to consider estimating effective cartel dates for the purpose of setting deterring fines.

Detailed knowledge of the cartel's price dynamics remains essential also to determine the likely direction of bias in estimating the cartel effects. During price wars that are part of the cartel spell, prices may be lower than in competition, as in Abreu et al. (1986) optimal temporary punishment strategies. Cyclical cartel price dynamics as in Rotemberg and Saloner (1986) can instead include low price episodes in which there is still a substantial overcharge—although gradual price movements in continuous collusion may not give rise to structural breaks, as in *Sodium Chlorate.* Including such intermittent episodes in the but-for price estimations can bias them, respectively down or up.

If there is indication that intermittent pricing is different from Nash-reversion, these episodes can be identified as separate regimes by appropriately including more than one dummy, as used in the structural break tests.[23] Likewise, if cost-elasticities are structurally higher in competition than collusion, or capacity management is tighter, an extended empirical cartel dating procedure can determine such additional cartel effects as parameter breaks between collusive and competitive regimes.

Alternatively, once the effective cartel dates have been determined, but-for prices can be predicted using the forecasting approach from data outside the effective cartel period(s), as well as any intermittent period(s) identified as other than competitive pricing. Properly dating the cartel assures that only data from non-cartel period(s) is used. Preliminary findings indicate that our results on misdating bias also apply to the forecasting approach, so that dated forecasting would minimize underestimation of total damages as well. We leave for future research too analysis of autoregressive dynamics when multiple effective cartel periods alternate with competitive episodes.

[23]Ellison (1994) revisits the data on price wars during the Joint Executive Committee railroad cartel analyzed in Porter (1983), to conclude that they were most likely of the Nash-reversion type explained in Green and Porter (1984).

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Cartel Dating
Online Appendix

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Appendix A: Proofs

Proof of Proposition 1. We first provide a detailed proof for $R = 1$ and then discuss the straightforward generalization to $R > 1$. Regarding the cartel dummy D_t we use the definition in (2). Following the standard asymptotic analysis of structural breaks (Perron, 1989), we assume that $T_B = \lambda_B T$ and $T_E = \lambda_E T$ with the break fractions λ_B and λ_E fixed numbers for all values of T.

We have for the average estimated but-for price

$$\operatorname{plim} \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} \widehat{bfp}_t = \operatorname{plim} \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} bfp_t + \operatorname{plim} \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} \left(\widehat{bfp}_t - bfp_t \right).$$

We will analyze the limiting behavior of the second right-hand side term in more detail. Noting that

$$\widehat{bfp}_{T_B} = bfp_{T_B} = p_{T_B},$$

in period $T_B + 1$ we have for the estimated and effective but-for prices:

$$\begin{aligned} \widehat{bfp}_{T_B+1} &= \hat{\gamma} p_{T_B} + \hat{\beta}' x_{T_B+1} + \hat{\alpha}_1, \\ bfp_{T_B+1} &= \gamma p_{T_B} + \beta' x_{T_B+1} + \alpha_1 + \varepsilon_{T_B+1}. \end{aligned}$$

Therefore, we can write for the prediction error

$$\begin{aligned} v_{T_B+1} &= \widehat{bfp}_{T_B+1} - bfp_{T_B+1} \\ &= (\hat{\gamma} - \gamma) p_{T_B} + (\hat{\beta} - \beta)' x_{T_B+1} + \hat{\alpha}_1 - \alpha_1 - \varepsilon_{T_B+1} \\ &= -\varepsilon_{T_B+1} + O_P(T^{-1/2}), \end{aligned}$$

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because estimation errors are $O_P(T^{-1/2})$ under Assumption 1 due to standard asymptotic theory. In period T_B+2 we have for the difference in estimated and effective but-for prices

$$\begin{aligned}
\widehat{bfp}_{T_B+2} - bfp_{T_B+2} &= \hat{\gamma}\widehat{bfp}_{T_B+1} - \gamma bfp_{T_B+1} + (\hat{\beta}-\beta)'x_{T_B+2} + \hat{\alpha}_1 - \alpha_1 - \varepsilon_{T_B+2} \\
&= \hat{\gamma}v_{T_B+1} + (\hat{\gamma}-\gamma)\, bfp_{T_B+1} + (\hat{\beta}-\beta)'x_{T_B+2} + \hat{\alpha}_1 - \alpha_1 - \varepsilon_{T_B+2} \\
&= \hat{\gamma}v_{T_B+1} + v_{T_B+2},
\end{aligned}$$

where for the prediction error v_{T_B+2} we have

$$v_{T_B+2} = -\varepsilon_{T_B+2} + O_P(T^{-1/2}).$$

In general, we have for $s = 1, 2, \ldots, T_E - T_B$

$$\begin{aligned}
\widehat{bfp}_{T_B+s} - bfp_{T_B+s} &= \hat{\gamma}^{s-1}v_{T_B+1} + \hat{\gamma}^{s-2}v_{T_B+2} + \ldots + \hat{\gamma}v_{T_B+s-1} + v_{T_B+s} \\
&= \sum_{j=0}^{s-1} \gamma^j v_{T_B+s-j} + O_P(T^{-1/2}) \\
&= -\sum_{j=0}^{s-1} \gamma^j \varepsilon_{T_B+s-j} + O_P(T^{-1/2}),
\end{aligned}$$

where the second line follows from $\hat{\gamma} = \gamma + O_P(T^{-1/2})$. Therefore,

$$\operatorname{plim} \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} \left(\widehat{bfp}_t - bfp_t\right) = -\operatorname{plim} \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} \sum_{j=0}^{s-1} \gamma^j \varepsilon_{T_B+s-j} = 0,$$

by Chebyshev's inequality, using $E[\varepsilon_t] = 0$ and $E[\varepsilon_t \varepsilon_s] = \sigma_N^2 + D_t(\sigma_C^2 - \sigma_N^2)$ for $s = t$, and 0 otherwise. This completes the proof for $R = 1$ because, due to stationarity (Assumption 1) we have

$$\operatorname{plim} \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} bfp_t = E\left[bfp_t\right].$$

Consider next multiple effective cartel periods ($R > 1$), i.e. $\mathbb{T}_C = \mathbb{T}_{C_1} \cup \ldots \cup \mathbb{T}_{C_R}$. Each set of effective cartel periods $\mathbb{T}_{C_r}$, $r = 1, \ldots, R$, has a known begin date T_{B_r} and end date T_{E_r}, so that $\mathbb{T}_{C_r} = \{T_{B_r}+1, \ldots, T_{E_r}\}$ with period length $T_{C_r} = T_{E_r} - T_{B_r}$. We assume that $T_{C_r} = \lambda_{C_r} T$ with the break fraction λ_{C_r} a fixed number for all values of T. The above derivations are then valid for each period, and we have

$$\begin{aligned}
\operatorname{plim} \frac{1}{T_C} \sum_{t \in \mathbb{T}_C} \widehat{bfp}_t &= \sum_{r=1}^{R} \lim \frac{T_{C_r}}{T_C} \operatorname{plim} \frac{1}{T_{C_r}} \sum_{t \in \mathbb{T}_{C_r}} \widehat{bfp}_t \\
&= \sum_{r=1}^{R} \frac{\lambda_{C_r}}{\lambda_C} \operatorname{plim} \frac{1}{T_{C_r}} \sum_{t \in \mathbb{T}_{C_r}} bfp_t \\
&= E\left[bfp_t\right],
\end{aligned}$$

which completes the proof. □

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Proof of Proposition 2. We first provide a detailed proof for $R = 1$ and then discuss the straightforward generalization to $R > 1$. Noting that $bfp_{T_B} = p_{T_B}$ we have for the overcharge

$$O_{T_B+s} = (p_{T_B+s} - bfp_{T_B+s}) = \frac{1-\gamma^s}{1-\gamma}\alpha_2,$$

for $s = 1, 2, \ldots, T_E - T_B$. Therefore, we have for the average effective overcharge

$$\begin{aligned}
\text{plim}\,\bar{O} &= \text{plim}\,\frac{1}{T_E - T_B}\sum_{t=T_B+1}^{T_E}(p_t - bfp_t) \\
&= \lim \frac{1}{T_E - T_B}\sum_{s=1}^{T_E-T_B}\frac{1-\gamma^s}{1-\gamma}\alpha_2 \\
&= \frac{\alpha_2}{1-\gamma}.
\end{aligned}$$

Using $\widehat{O}_{1t}$ we have for the average estimated overcharge

$$\begin{aligned}
\text{plim}\,\overline{O}_1 &= \text{plim}\,\frac{1}{T_E - T_B}\sum_{s=1}^{T_E-T_B}\frac{1-\hat{\gamma}^s}{1-\hat{\gamma}}\hat{\alpha}_2 \\
&= \text{plim}\,\frac{\hat{\alpha}_2}{1-\hat{\gamma}}\left(1 - \text{plim}\,\frac{1}{T_E - T_B}\sum_{s=1}^{T_E-T_B}\hat{\gamma}^s\right) \\
&= \frac{\alpha_2}{1-\gamma}.
\end{aligned}$$

Using $\widehat{O}_{2t}$ we have for the average estimated overcharge

$$\begin{aligned}
\text{plim}\,\overline{O}_2 &= \text{plim}\,\frac{1}{T_E - T_B}\sum_{t=T_B+1}^{T_E}\left(p_t - \widehat{bfp}_t\right) \\
&= \text{plim}\,\frac{1}{T_E - T_B}\sum_{t=T_B+1}^{T_E}(p_t - bfp_t) - \text{plim}\,\frac{1}{T_E - T_B}\sum_{t=T_B+1}^{T_E}\left(\widehat{bfp}_t - bfp_t\right) \\
&= \text{plim}\,\bar{O},
\end{aligned}$$

which follows directly from Proposition 1.

Consider next multiple periods of cartel effects ($R > 1$). The above derivations are valid for each period and the probability limit of the overcharge $\frac{\alpha_2}{1-\gamma}$ is constant for each period. Therefore, when $R > 1$ we simply have

$$\text{plim}\,\bar{O} = \sum_{r=1}^{R}\frac{\lambda_{C_r}}{\lambda_C}\frac{\alpha_2}{1-\gamma} = \frac{\alpha_2}{1-\gamma},$$

and similarly for $\bar{O}_1$ and $\bar{O}_2$. This completes the proof. □

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Assumption $2(ii)$ **under Linear Demand.** Suppose model (1) and Assumption 1 hold. Consider the stylized linear demand function

$$Q_t = a + bp_t + \epsilon_t,$$

where $a > 0$ and ϵ_t is an error term with $E[\epsilon_t|p_t] = 0$. Then Assumption 2 (ii) holds when demand is downward sloping ($b \leq 0$) and x_t is strictly exogenous, i.e. $E[x_t\varepsilon_s] = 0$ for all $s, t = 1, \ldots, T$.

From model (1),

$$p_t = \frac{\alpha_1}{1-\gamma} + \alpha_2 \sum_{i=0}^{\infty} \gamma^i D_{t-i} + \beta \sum_{i=0}^{\infty} \gamma^i x_{t-i} + \sum_{i=0}^{\infty} \gamma^i \varepsilon_{t-i},$$

so that

$$Q_t = a + \frac{b\alpha_1}{1-\gamma} + b\alpha_2 \sum_{i=0}^{\infty} \gamma^i D_{t-i} + b\beta \sum_{i=0}^{\infty} \gamma^i x_{t-i} + b \sum_{i=0}^{\infty} \gamma^i \varepsilon_{t-i} + \epsilon_t.$$

Under Assumption 1 and strict exogeneity of x_t,

$$\begin{aligned} E\left[Q_t \varepsilon_{t-j}\right] &= E\left[\left(b\beta \sum_{i=0}^{\infty} \gamma^i x_{t-i} + b \sum_{i=0}^{\infty} \gamma^i \varepsilon_{t-i} + \epsilon_t\right) \varepsilon_{t-j}\right] \\ &= b\gamma^j E[\varepsilon_{t-j}^2] \\ &\leq 0, \end{aligned}$$

for $b \leq 0$, with equality if and only if $b = 0$, as $0 < \gamma < 1$.

The example above assumes $E[\epsilon_t|p_t] = 0$, which effectively says that demand shocks ϵ_t do not affect prices p_t. When $E[\epsilon_t|p_t] \neq 0$, Assumption 2 (ii) may still hold under weak additional conditions, as the following example illustrates.[1] Consider the symmetric Cournot oligopoly model. The inverse demand function is

$$p_t = e - fQ_t + u_{pt},$$

where $e = -a/b$, $f = -1/b$ and $u_{pt} = -\epsilon_t/b$. Suppose that the marginal costs of production have mean c and can be expressed as $c + u_{ct}$, with u_{ct} a random cost shock. For ease of exposition, assume that the demand and cost shocks u_{pt} and u_{ct} are uncorrelated, mean-zero random variables with variances σ_p^2 and σ_c^2, respectively. Then the Cournot (competitive) equilibrium with n firms implies

$$p_{nt} = \frac{e + nc + nu_{ct} + u_{pt}}{n+1},$$

$$Q_{nt} = \frac{n}{n+1} \frac{e - c + u_{pt} - u_{ct}}{f},$$

[1]We are indebted to one of the anonymous referees for suggesting this illustrative example.

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with p_{nt} and Q_{nt} the equilibrium prices and quantities, respectively. If we furthermore assume the cartel to behave as a monopolist, the joint-profit maximizing cartel (monopoly) price and quantity are

$$p_{mt} = \frac{e + c + u_{ct} + u_{pt}}{2},$$
$$Q_{mt} = \frac{e - c + u_{pt} - u_{ct}}{2f}.$$

In this market, the observed price is the Cournot (competitive) price p_{nt} if $D_t = 0$, and the cartel (monopoly) price p_{mt} if $D_t = 1$, that is

$$p_t = p_{nt} + D_t(p_{mt} - p_{nt}) = \begin{cases} p_{nt}, & D_t = 0, \\ p_{mt}, & D_t = 1. \end{cases}$$

The Cournot price can be written as

$$p_{nt} = \alpha_1 + \eta_t, \qquad \alpha_1 = \frac{e + nc}{n+1}, \qquad \eta_t = \frac{u_{pt} + nu_{ct}}{n+1},$$

while the difference between monopoly prices and Cournot prices, i.e. the overcharge, is

$$p_{mt} - p_{nt} = \alpha_2 + \upsilon_t, \qquad \alpha_2 = \frac{n-1}{2(n+1)}(e - c), \quad \upsilon_t = \frac{(n-1)(u_{pt} - u_{ct})}{2(n+1)}.$$

Summarizing, we can write for the observed price

$$\begin{aligned} p_t &= \alpha_1 + \alpha_2 D_t + \eta_t + D_t \upsilon_t \\ &= \alpha_1 + \alpha_2 D_t + \varepsilon_t, \end{aligned}$$

which is a stripped down version of model (1). Note that

$$\sigma_N^2 = \text{Var}\left(\varepsilon_t | D_t = 0\right) = \text{Var}(\eta_t) = \frac{\sigma_p^2 + n^2\sigma_c^2}{(n+1)^2},$$
$$\sigma_C^2 = \text{Var}\left(\varepsilon_t | D_t = 1\right) = \text{Var}(\eta_t + \upsilon_t) = \frac{\sigma_p^2 + \sigma_c^2}{4},$$

which implies different variances during cartel and non-cartel periods, in agreement with Assumption 1. Furthermore, we have

$$\text{Cov}\left(Q_t, \varepsilon_t | D_t = 0\right) = \text{Cov}\left(Q_{nt}, \eta_t\right) = \frac{n\left(\sigma_p^2 - n\sigma_c^2\right)}{f(n+1)^2},$$
$$\text{Cov}\left(Q_t, \varepsilon_t | D_t = 1\right) = \text{Cov}\left(Q_{mt}, \eta_t + \upsilon_t\right) = \frac{\sigma_p^2 - \sigma_c^2}{4f}.$$

The sign of these covariances clearly depends on the relative magnitude of the variances of the (inverse) demand shocks and cost shocks. When $\sigma_c^2 > \sigma_p^2$, Assumption 2 (*ii*) is satisfied. Analogously, specific additional conditions can be formulated for more complex models of oligopolistic competition. □

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Proof of Theorem 1. We first provide a detailed proof for $R = 1$ and then discuss the straightforward generalization to $R > 1$. For the effective damage, we have

$$\begin{aligned}
\operatorname{plim} \frac{1}{T} CD &= \operatorname{plim} \frac{1}{T} \sum_{t=T_B+1}^{T_E} (p_t - bfp_t)\, Q_t \\
&= \operatorname{plim} \frac{1}{T} \sum_{s=1}^{T_E-T_B} \frac{1-\gamma^s}{1-\gamma} \alpha_2 Q_{T_B+s} \\
&= \frac{\alpha_2}{1-\gamma} \left(\frac{T_E - T_B}{T} \operatorname{plim} \frac{1}{T_E - T_B} \sum_{s=1}^{T_E-T_B} Q_{T_B+s} - \frac{1}{T} \sum_{s=1}^{T_E-T_B} \gamma^s Q_{T_B+s} \right) \\
&= \frac{\alpha_2}{1-\gamma} (\lambda_E - \lambda_B)\, Q_C. \qquad \text{(A.1)}
\end{aligned}$$

The final result follows from (15), together with the fact that $\sum_{s=1}^{T_E-T_B} \gamma^s Q_{T_B+s} = O_P(1)$.

Using $\widehat{O}_{1t}$ we have for the average estimated damage

$$\begin{aligned}
\operatorname{plim} \frac{1}{T} \widehat{CD}_1 &= \operatorname{plim} \frac{1}{T_E - T_B} \sum_{s=1}^{T_E-T_B} \frac{1-\hat{\gamma}^s}{1-\hat{\gamma}} \hat{\alpha}_2 Q_{T_B+s} \\
&= \frac{\alpha_2}{1-\gamma} (\lambda_E - \lambda_B)\, Q_C,
\end{aligned}$$

which follows directly (A.1) and consistency of $\hat{\gamma}$. Using $\widehat{O}_{2t}$ we have for the average estimated damage

$$\begin{aligned}
\operatorname{plim} \frac{1}{T} \widehat{CD}_2 &= \operatorname{plim} \frac{1}{T} \sum_{t=T_B+1}^{T_E} \left(p_t - \widehat{bfp}_t \right) Q_t \\
&= \operatorname{plim} \frac{1}{T} \sum_{t=T_B+1}^{T_E} (p_t - bfp_t)\, Q_t + \operatorname{plim} \frac{1}{T} \sum_{t=T_B+1}^{T_E} \left(bfp_t - \widehat{bfp}_t \right) Q_t \\
&= \operatorname{plim} \frac{1}{T} CD - \operatorname{plim} \frac{1}{T} \sum_{s=1}^{T_E-T_B} \left(\widehat{bfp}_{T_B+s} - bfp_{T_B+s} \right) Q_{T_B+s}.
\end{aligned}$$

We will analyze the limiting behavior of the second term in more detail under Assumption 2. Proposition 1 implies that

$$\left(\widehat{bfp}_{T_B+s} - bfp_{T_B+s} \right) Q_{T_B+s} = - \sum_{j=0}^{s-1} \gamma^j \varepsilon_{T_B+s-j} Q_{T_B+s} + O_P(T^{-1/2}).$$

Assumption 2 implies that

$$- \sum_{j=0}^{s-1} E \left[\gamma^j \varepsilon_{T_B+s-j} Q_{T_B+s} \right] \geq 0,$$

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with equality if and only if Assumption 2 (ii) holds with equality. Applying a LLN we find

$$\operatorname{plim} \frac{1}{T} \sum_{s=1}^{T_E - T_B} \left(\widehat{bfp}_{T_B+s} - bfp_{T_B+s} \right) Q_{T_B+s} \geq 0,$$

again with equality if and only if Assumption 2 (ii) holds with equality.

When there are multiple periods of cartel effects ($R > 1$), the above derivations are valid for each period. When $R > 1$ we have

$$\begin{aligned} \operatorname{plim} \frac{1}{T} CD &= \sum_{r=1}^{R} \operatorname{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_{C_r}} (p_t - bfp_t) Q_t \\ &= \sum_{r=1}^{R} \frac{\alpha_2}{1-\gamma} \lambda_{C_r} Q_C = \frac{\alpha_2}{1-\gamma} \lambda_C Q_C, \end{aligned}$$

because $\lambda_C = \sum_{r=1}^{R} \lambda_{C_r}$. Because for each period $\widehat{CD}_1$ is consistent for CD, we have

$$\operatorname{plim} \frac{1}{T} \widehat{CD}_1 = \operatorname{plim} \frac{1}{T} CD.$$

Regarding $\widehat{CD}_2$ the weak inequality holds for each period, hence

$$\operatorname{plim} \frac{1}{T} \widehat{CD}_2 \leq \operatorname{plim} \frac{1}{T} CD,$$

which completes the proof. □

Proof of Lemma 1. It is helpful for further calculations on the OLS inconsistency to centre all regressors such they have mean zero. The reason is that we have a constant term in the model, so all variables can be taken in deviation from their sample average, affecting only the definition and interpretation of the intercept. If we let x_t^0 denote the original control variable, we then have

$$x_t = x_t^0 - \bar{x}^0,$$

such that

$$m_x = \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} x_t = 0.$$

Moreover, we can redefine the cartel dummy variables D_t and d_t such that actually the measurement error $v_t = d_t - D_t$ has mean zero. If we let D_t^0 and d_t^0 denote the original 0-1 dummy variables, we define D_t and d_t as dummy variables in deviation from their sample mean:

$$\begin{aligned} D_t &= D_t^0 - (\lambda_{01} + \lambda_{11}), \\ d_t &= d_t^0 - (\lambda_{10} + \lambda_{11}). \end{aligned}$$

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The result is that, irrespective of the type of misclassification, the average measurement error is zero, i.e.

$$\frac{1}{T}\sum_{t=1}^{T} v_t = 0.$$

The reparametrized model is

$$\begin{aligned} p_t &= \alpha_1 + \alpha_2 D_t^0 + \beta x_t^0 + \varepsilon_t \\ &= \alpha + \alpha_2 D_t + \beta x_t + \varepsilon_t, \end{aligned}$$

so that the new intercept becomes

$$\alpha = \alpha_1 + (\lambda_{01} + \lambda_{11})\,\alpha_2 + \beta\bar{x}^0.$$

Analogously, the estimated model becomes

$$\begin{aligned} \hat{p}_t &= \hat{\alpha}_1 + \hat{\alpha}_2 d_t^0 + \hat{\beta} x_t^0 \\ &= \hat{\alpha} + \hat{\alpha}_2 d_t + \hat{\beta} x_t, \end{aligned}$$

with

$$\hat{\alpha} = \hat{\alpha}_1 + (\lambda_{10} + \lambda_{11})\,\hat{\alpha}_2 + \hat{\beta}\bar{x}^0.$$

Stacking the observations ($t = 1, \ldots, T$), we write the regression model to be estimated as

$$y = Z\theta + u,$$

where $y = (p_1, \ldots, p_T)'$ and $u = (u_1, \ldots, u_T)'$. Furthermore, $Z = (z_1, \ldots, z_T)'$ with $z_t = (x_t, 1, d_t)'$ and $\theta = (\beta, \alpha, \alpha_2)'$. The OLS estimator of the full parameter vector θ is equal to

$$\hat{\theta} = (Z'Z)^{-1}Z'y.$$

Taking the probability limit we have

$$\begin{aligned} \operatorname{plim}\hat{\theta} &= \theta + \left(\operatorname{plim}\frac{1}{T}Z'Z\right)^{-1} \operatorname{plim}\frac{1}{T}Z'u \\ &= \theta + \Sigma_{ZZ}^{-1}\Sigma_{Zu}. \end{aligned} \tag{A.2}$$

The vector $\Sigma_{ZZ}^{-1}\Sigma_{Zu}$ is the OLS inconsistency.

Regarding Σ_{Zu} in (A.2) we have:

$$\begin{aligned} m_{xu} &= \operatorname{plim}\frac{1}{T}\sum_{t=1}^{T} x_t u_t = 0, \\ m_u &= \operatorname{plim}\frac{1}{T}\sum_{t=1}^{T} u_t = 0, \end{aligned}$$

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where $u_t = \varepsilon_t - \alpha_2 v_t$. Also we have

$$E\left[d_t u_t\right] = -\alpha_2 E\left[d_t v_t\right] \neq 0,$$

hence $m_{du} \neq 0$. Regarding Σ_{ZZ} in (A.2) we furthermore have:

$$m_x = m_d = 0,$$
$$m_{xd} = 0,$$

where the last equality follows from Assumption 1. Under the assumptions, we have now:

$$\Sigma_{ZZ} = \begin{pmatrix} m_{xx} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & m_{dd} \end{pmatrix}, \qquad \Sigma_{Zu} = \begin{pmatrix} 0 \\ 0 \\ m_{du} \end{pmatrix},$$

hence the inconsistency simply becomes

$$\Sigma_{ZZ}^{-1}\Sigma_{Zu} = \begin{pmatrix} 0 \\ 0 \\ \frac{m_{du}}{m_{dd}} \end{pmatrix}.$$

The inconsistency for α and β is zero, while it is $\frac{m_{du}}{m_{dd}}$ for α_2. Evaluating the latter we have

$$\begin{aligned} m_{du} &= \operatorname{plim} \frac{1}{T}\sum_{t=1}^{T} d_t u_t \\ &= \operatorname{plim} \frac{1}{T}\sum_{t=1}^{T} d_t \left(\varepsilon_t - \alpha_2 \left(d_t - D_t\right)\right) \\ &= -\alpha_2 \left(m_{dd} - m_{dD}\right), \end{aligned}$$

hence we have

$$\begin{aligned} \operatorname{plim} \hat{\alpha}_2 &= \alpha_2 \frac{m_{dD}}{m_{dd}} \\ &= \alpha_2 \left(\frac{\lambda_{11} - \left(\lambda_{10} + \lambda_{11}\right)\left(\lambda_{01} + \lambda_{11}\right)}{\left(\lambda_{10} + \lambda_{11}\right)\left(1 - \lambda_{10} - \lambda_{11}\right)} \right). \end{aligned}$$

Define:

$$a = \lambda_{11}, \quad b = \lambda_{10} + \lambda_{11}, \quad c = \lambda_{01} + \lambda_{11},$$

then the attenuation bias $\text{AB} = \operatorname{plim} \hat{\alpha}_2 / \alpha_2$ becomes

$$\text{AB} = \frac{a - bc}{b(1-b)}.$$

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We have $0 < b < 1$, hence $b(1-b) > 0$. Furthermore, because $0 \leq a \leq c < 1$ we have that

$$\text{AB} = \frac{a - bc}{b(1-b)} \leq \frac{a - ba}{b(1-b)} = \frac{a}{b} \leq 1.$$

The equality signs only hold when both $\lambda_{01} = 0$ and $\lambda_{10} = 0$. Therefore with misdating we have that $\text{AB} < 1$, hence $\text{plim}\,\hat{\alpha}_2 < \alpha_2$.

Regarding $\hat{\alpha}_1$ we have

$$\begin{aligned}
\text{plim}\,\hat{\alpha}_1 &= \alpha_1 + \text{plim}\,(\hat{\alpha} - \alpha) - (\lambda_{10} + \lambda_{11})\,\text{plim}\,\hat{\alpha}_2 + (\lambda_{01} + \lambda_{11})\,\alpha_2 - \text{plim}\,\bar{x}^0\,\text{plim}\left(\hat{\beta} - \beta\right) \\
&= \alpha_1 - (\lambda_{10} + \lambda_{11})\,\text{plim}\,\hat{\alpha}_2 + (\lambda_{01} + \lambda_{11})\,\alpha_2 \\
&= \alpha_1 + \alpha_2 \frac{\lambda_{01}}{\lambda_{00} + \lambda_{01}}.
\end{aligned}$$

We therefore have

$$\text{plim}\,\hat{\alpha}_1 \geq \alpha_1,$$

with equality only when $\lambda_{01} = 0$. This completes the proof. □

Proof of Proposition 3.

(*i*) The average estimated but-for price for the DGP (16) is

$$\text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c}\widehat{bfp}_t = \text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c} bfp_t + \text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c}\left(\widehat{bfp}_t - bfp_t\right).$$

We will analyze the limiting behavior of the second right-hand side term in more detail. We can write for the probability limit of the average prediction error

$$\begin{aligned}
\text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c}\left(\widehat{bfp}_t - bfp_t\right) &= \text{plim}(\hat{\beta} - \beta)\,\text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c} x_t + \text{plim}\,(\hat{\alpha}_1 - \alpha_1) - \text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c}\varepsilon_t \\
&= \text{plim}\,(\hat{\alpha}_1 - \alpha_1)\,,
\end{aligned}$$

noting that Lemma 1 proofs that $\text{plim}(\hat{\beta} - \beta) = 0$. The result now follows straightforwardly from Lemma 1.

(*ii*) From Lemma 1 we know that $0 < \text{plim}\,\hat{\alpha}_2 < \alpha_2$ always from which the result for $\overline{O}_1$ follows. Using $\widehat{O}_{2t}$ we have for the average estimated overcharge

$$\begin{aligned}
\text{plim}\,\overline{O}_2 &= \frac{1}{T_c}\sum_{t\in\mathbb{T}_c}\left(p_t - \widehat{bfp}_t\right) \\
&= \text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c}(p_t - bfp_t) - \text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c}\left(\widehat{bfp}_t - bfp_t\right) \\
&\leq \text{plim}\,\frac{1}{T_c}\sum_{t\in\mathbb{T}_c}(p_t - bfp_t)\,,
\end{aligned}$$

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where the equality holds only when $\lambda_{01} = 0$. Furthermore, note that

$$\begin{aligned}
\operatorname{plim} \frac{1}{T_c} \sum_{t \in \mathbb{T}_c} (p_t - bfp_t) &= \frac{\lambda_{10}}{\lambda_{10} + \lambda_{11}} \operatorname{plim} \frac{1}{T_{10}} \sum_{t \in \mathbb{T}_{10}} (p_t - bfp_t) \\
&\quad + \frac{\lambda_{11}}{\lambda_{10} + \lambda_{11}} \operatorname{plim} \frac{1}{T_{11}} \sum_{t \in \mathbb{T}_{11}} (p_t - bfp_t) \\
&= \frac{\lambda_{11}}{\lambda_{10} + \lambda_{11}} \alpha_2 \\
&\leq \alpha_2,
\end{aligned}$$

where the equality holds only when $\lambda_{10} = 0$. Because in case of misdating λ_{01} and λ_{10} cannot be zero at the same time, we have

$$\operatorname{plim} \overline{O}_2 < \alpha_2.$$

This completes the proof. □

Proof of Theorem 2. We have for the effective damage

$$\begin{aligned}
\operatorname{plim} \frac{1}{T} CD &= \operatorname{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_C} (p_t - bfp_t)\, Q_t \\
&= \operatorname{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_C} \alpha_2 Q_t \\
&= \alpha_2 \left(\lambda_{01} + \lambda_{11}\right) Q_C.
\end{aligned}$$

Regarding the estimator $\widehat{CD}_1$ we have

$$\begin{aligned}
\operatorname{plim} \frac{1}{T} \widehat{CD}_1 &= \operatorname{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_c} \hat{\alpha}_2 Q_t \\
&= \operatorname{plim} \hat{\alpha}_2 \operatorname{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_c} Q_t \\
&= \alpha_2 \left(\frac{\lambda_{11} - \left(\lambda_{10} + \lambda_{11}\right)\left(\lambda_{01} + \lambda_{11}\right)}{\left(\lambda_{10} + \lambda_{11}\right)\left(1 - \lambda_{10} - \lambda_{11}\right)} \right) \\
&\quad \times \left(\lambda_{10} Q_N + \lambda_{11} Q_C\right).
\end{aligned}$$

Comparing $\widehat{CD}_1$ with CD, when $Q_N \neq Q_C$ no systematic pattern emerges, i.e.

$$\operatorname{plim} \frac{1}{T} \widehat{CD}_1 \gtreqless \operatorname{plim} \frac{1}{T} CD.$$

When $Q_N = Q_C$, however, some algebra shows that

$$\begin{aligned}
\operatorname{plim} \frac{1}{T} \widehat{CD}_1 &= \operatorname{plim} \frac{1}{T} CD - \alpha_2 \frac{\lambda_{01}}{1 - \lambda_{10} - \lambda_{11}} Q_C \\
&\leq \operatorname{plim} \frac{1}{T} CD.
\end{aligned}$$

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When $\lambda_{01} = 0$ we furthermore have

$$\begin{aligned}
\text{plim}\,\frac{1}{T}\widehat{CD}_1 &= \alpha_2 \frac{\lambda_{11}}{\lambda_{10} + \lambda_{11}} \left(\lambda_{10} Q_N + \lambda_{11} Q_C\right) \\
&\geq \alpha_2 \lambda_{11} Q_C \\
&= \text{plim}\,\frac{1}{T} CD,
\end{aligned}$$

where the exact equality holds when $Q_N = Q_C$ too. When $\lambda_{10} = 0$ we have

$$\begin{aligned}
\text{plim}\,\frac{1}{T}\widehat{CD}_1 &= \alpha_2 \lambda_{11} \frac{1 - \lambda_{01} - \lambda_{11}}{1 - \lambda_{11}} Q_C \\
&< \alpha_2 \lambda_{11} Q_C \\
&< \text{plim}\,\frac{1}{T} CD.
\end{aligned}$$

Regarding the $\widehat{CD}_2$ estimator we have

$$\begin{aligned}
\text{plim}\,\frac{1}{T}\widehat{CD}_2 &= \text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_c} \left(p_t - \widehat{bfp}_t\right) Q_t \\
&= \text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_c} \left(p_t - bfp_t\right) Q_t - \text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_c} \left(\widehat{bfp}_t - bfp_t\right) Q_t.
\end{aligned}$$

We will analyze the limiting behavior of both the first and second terms in more detail. Regarding the first term we have

$$\begin{aligned}
\text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_c} \left(p_t - bfp_t\right) Q_t &= \text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_C} \left(p_t - bfp_t\right) Q_t \\
&\quad + \text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_{10}} \left(p_t - bfp_t\right) Q_t - \text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_{01}} \left(p_t - bfp_t\right) Q_t \\
&= \text{plim}\,\frac{1}{T} CD - \alpha_2 \lambda_{01} Q_C \\
&\leq \text{plim}\,\frac{1}{T} CD,
\end{aligned}$$

where equality holds only if $\lambda_{01} = 0$. Regarding the second term we have

$$\widehat{bfp}_t - bfp_t = \left(\hat{\alpha}_1 - \alpha_1\right) + \left(\hat{\beta} - \beta\right) x_t - \varepsilon_t,$$

and using the results from Lemma 1 and Assumption 2 we can write:

$$\begin{aligned}
\text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_c} \left(\hat{\alpha}_1 - \alpha_1\right) Q_t &= \text{plim}\left(\hat{\alpha}_1 - \alpha_1\right) \text{plim}\,\frac{1}{T}\sum_{t \in \mathbb{T}_c} Q_t \\
&= \alpha_2 \frac{\lambda_{01}}{\lambda_{00} + \lambda_{01}} \left(\lambda_{10} Q_N + \lambda_{11} Q_C\right) \\
&\geq 0,
\end{aligned}$$

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$$\text{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_c} \left(\hat{\beta} - \beta\right) x_t Q_t = \text{plim} \left(\hat{\beta} - \beta\right) \text{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_c} x_t Q_t$$
$$= 0,$$

$$\text{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_c} \varepsilon_t Q_t \leq 0.$$

Collecting terms we find

$$\text{plim} \frac{1}{T} \sum_{t \in \mathbb{T}_c} \left(\widehat{bfp}_t - bfp_t\right) Q_t \geq 0.$$

Equality results only when price and quantity are uncorrelated and $\lambda_{01} = 0$. In all other cases a strict inequality follows. This completes the proof. □

Proof of Lemma 2. It is helpful for further calculations on the OLS inconsistency to invoke the result that without loss of generalization we can assume that x_t has mean zero, hence

$$m_x = \text{plim} \frac{1}{T} \sum_{t=1}^{T} x_t = 0.$$

Moreover, we can redefine the cartel dummy variable such that actually the measurement error $v_t = d_t - D_t$ has mean zero. If we let D_t^0 and d_t^0 denote the original $0 - 1$ dummy variables, such that $D_t = D_t^0 - (\lambda_E - \lambda_B)$ and $d_t = d_t^0 - (\lambda_e - \lambda_b)$, then

$$\begin{aligned} p_t &= \alpha_1 + \alpha_2 D_t^0 + \beta' x_t + \gamma p_{t-1} + \varepsilon_t \\ &= \alpha + \alpha_2 D_t + \beta' x_t + \gamma p_{t-1} + \varepsilon_t, \end{aligned}$$

so that the new intercept becomes

$$\alpha = \alpha_1 + (\lambda_E - \lambda_B)\, \alpha_2.$$

Analogously, the estimated model becomes

$$\begin{aligned} \hat{p}_t &= \hat{\alpha}_1 + \hat{\alpha}_2 d_t^0 + \hat{\beta}' x_t + \hat{\gamma} p_{t-1} \\ &= \hat{\alpha} + \hat{\alpha}_2 d_t + \hat{\beta}' x_t + \hat{\gamma} p_{t-1}, \end{aligned}$$

with

$$\hat{\alpha} = \hat{\alpha}_1 + (\lambda_e - \lambda_b)\, \hat{\alpha}_2.$$

Stacking the observations ($t = 1, \ldots, T$), we write the regression model to be estimated as

$$y = Z\theta + u,$$

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where $y = (p_1, \ldots, p_T)'$ and $u = (u_1, \ldots, u_T)'$. Furthermore, $Z = (z_1, \ldots, z_T)'$ with $z_t = (p_{t-1}, x_t, 1, d_t)'$ and $\theta = (\gamma, \beta, \alpha, \alpha_2)'$. The OLS estimator of the full parameter vector θ is equal to

$$\hat{\theta} = (Z'Z)^{-1}Z'y.$$

Taking the probability limit we have

$$\begin{aligned} \text{plim}\,\hat{\theta} &= \theta + \left(\text{plim}\,\frac{1}{T}Z'Z\right)^{-1} \text{plim}\,\frac{1}{T}Z'u \\ &= \theta + \Sigma_{ZZ}^{-1}\Sigma_{Zu}. \end{aligned} \tag{A.3}$$

The vector $\Sigma_{ZZ}^{-1}\Sigma_{Zu}$ is the OLS inconsistency.

Regarding Σ_{Zu} in (A.3) we have:

$$m_{xu} = \text{plim}\,\frac{1}{T}\sum_{t=1}^{T} x_t u_t = 0,$$

$$m_u = \text{plim}\,\frac{1}{T}\sum_{t=1}^{T} u_t = 0,$$

where $u_t = \varepsilon_t - \alpha_2 v_t$. Also we have

$$E\left[d_t u_t\right] = -\alpha_2 E\left[d_t v_t\right] \neq 0,$$

hence $m_{du} \neq 0$. Also there is a non-zero correlation between p_{t-1} and u_t. Exploiting the stationarity assumption by repeated substitution we can write

$$p_t = \beta\sum_{s=0}^{\infty}\gamma^s x_{t-s} + \frac{\alpha}{1-\gamma} + \alpha_2\sum_{s=0}^{\infty}\gamma^s D_{t-s} + \sum_{s=0}^{\infty}\gamma^s \varepsilon_{t-s},$$

so we have (exploiting the fact that ε_t has no autocorrelation, and zero correlation with lagged x_{t-s-1} by Assumption 1) that

$$\begin{aligned} E\left[p_{t-1}u_t\right] &= -\alpha_2^2 E\left[v_t\sum_{s=0}^{\infty}\gamma^s D_{t-1-s}\right] \\ &= -\alpha_2^2\left(v_t D_{t-1} + \gamma v_t D_{t-2} + \gamma^2 v_t D_{t-3} + \ldots\right) \\ &\neq 0, \end{aligned}$$

so $m_{p_{-1}u} \neq 0$. Under the assumptions, we have now:

$$\Sigma_{ZZ} = \begin{pmatrix} m_{pp} & m_{xp_{-1}} & m_p & m_{dp_{-1}} \\ m_{xp_{-1}} & m_{xx} & 0 & 0 \\ m_p & 0 & 1 & 0 \\ m_{dp_{-1}} & 0 & 0 & m_{dd} \end{pmatrix}, \qquad \Sigma_{Zu} = \begin{bmatrix} m_{p_{-1}u} \\ 0 \\ 0 \\ m_{du} \end{bmatrix}.$$

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After some algebra we find for the inconsistency

$$\Sigma_{ZZ}^{-1}\Sigma_{Zu} = \frac{1}{\det\left(\Sigma_{ZZ}\right)} \left[\begin{array}{c} m_{xx}\left(m_{dd}m_{p_{-1}u} - m_{du}m_{dp_{-1}}\right) \\ -m_{xp_{-1}}\left(m_{dd}m_{p_{-1}u} - m_{du}m_{dp_{-1}}\right) \\ -m_p m_{xx}\left(m_{dd}m_{p_{-1}u} - m_{du}m_{dp_{-1}}\right) \\ -m_{dp_{-1}}m_{xx}m_{p_{-1}u} - m_{du}(\sigma_{xp_{-1}}^2 - \sigma_x^2\sigma_p^2) \end{array} \right], \tag{A.4}$$

where $\det\left(\Sigma_{ZZ}\right) > 0$ and we define $\sigma_p^2 = m_{pp} - m_p^2$. Furthermore, because we assume $m_x = 0$, we have $m_{xx} = \sigma_x^2$ and $m_{xp_{-1}} = \sigma_{xp_{-1}}$.

We now have to evaluate all separate terms in (A.4). We always have

$$\begin{aligned} m_{dd} &= \operatorname{plim} \frac{1}{T} \sum d_t^2 \\ &= \left(1 - \lambda_e + \lambda_b\right)\left(\lambda_e - \lambda_b\right). \end{aligned}$$

Furthermore, to evaluate $m_{p_{-1}u}$ we note that

$$\begin{aligned} m_{p_{-1}u} &= \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} p_{t-1}u_t \\ &= -\alpha_2^2 \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} \left(v_t D_{t-1} + \gamma v_t D_{t-2} + \gamma^2 v_t D_{t-3} + \ldots\right) \\ &= -\alpha_2^2 \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} \sum_{s=0}^{\infty} \gamma^s v_t D_{t-1-s} \\ &= -\alpha_2^2 \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} \sum_{s=0}^{\infty} \gamma^s v_t \left(D_t + D_{t-1-s} - D_t\right) \\ &= -\frac{\alpha_2^2}{1-\gamma} \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} v_t D_t + \alpha_2^2 \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} \sum_{s=0}^{\infty} \gamma^s v_t \left(D_t - D_{t-1-s}\right) \\ &= -\frac{\alpha_2^2}{1-\gamma} \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} v_t D_t. \end{aligned}$$

The final equality holds because $v_t\left(D_t - D_{t-1-s}\right) \propto \frac{s+1}{T}$, because $D_t - D_{t-1-s}$ has nonzero values in (two times) $\frac{s+1}{T}$ observations only. Therefore,

$$\begin{aligned} \sum_{s=0}^{\infty} \gamma^s v_t \left(D_t - D_{t-1-s}\right) &\propto \sum_{s=0}^{\infty} \gamma^s \frac{s+1}{T} \\ &= \frac{1}{T} \sum_{s=0}^{\infty} \left(s+1\right)\gamma^s \\ &= \frac{1}{T\left(1-\gamma\right)^2}, \end{aligned}$$

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which is of order $O(T^{-1})$ only.

For the same reason we have

$$\begin{aligned} m_{dp_{-1}} &= \text{plim}\, \frac{1}{T} \alpha_2 \sum_{t=1}^{T} \sum_{s=0}^{\infty} \gamma^s d_t D_{t-1-s} \\ &= \alpha_2 \, \text{plim}\, \frac{1}{T} \sum_{t=1}^{T} \sum_{s=0}^{\infty} \gamma^s d_t \left(D_t - (D_t - D_{t-1-s}) \right) \\ &= \frac{\alpha_2}{1-\gamma} \, \text{plim}\, \frac{1}{T} \sum_{t=1}^{T} d_t D_t. \end{aligned}$$

The precise magnitude of the separate terms will depend on the type of break misdating. The four possible misdating scenarios for the break dates are:

$$\begin{aligned} &\text{Case 1: } T_b < T_B, \; T_e < T_E \\ &\text{Case 2: } T_b < T_B, \; T_e > T_E \\ &\text{Case 3: } T_b > T_B, \; T_e < T_E \\ &\text{Case 4: } T_b > T_B, \; T_e > T_E \end{aligned} \tag{A.5}$$

In Case 1 the cartel is formally dated to begin and end too early—including a formal cartel period that entirely preceeds the cartel's effects. In Case 2 the formal cartel period encompasses the effective cartel period and includes non-cartel periods too. Case 3 is a legally too narrowly defined period, for which there is indication competition authorities conservatively do. Case 4 is the mirror image of Case 1. Compared to the misclassification set up introduced in (18), Case 2 is equal to the special situation that $\lambda_{01} = 0$, while Case 3 amounts to $\lambda_{10} = 0$. For Cases 1 and 4 both $\lambda_{01} > 0$ and $\lambda_{10} > 0$.

We will provide detailed derivations for Cases 1 and 2 only. In Case 1 ($T_b < T_B < T_e < T_E$) we have for the measurement error:

$$v_t = \begin{cases} \lambda_b - \lambda_e - \lambda_B + \lambda_E, & t \leq T_b, \\ 1 - \lambda_e + \lambda_b - \lambda_B + \lambda_E, & T_b < t \leq T_B, \\ \lambda_b - \lambda_e - \lambda_B + \lambda_E, & T_B < t \leq T_e, \\ \lambda_b - \lambda_e - 1 - \lambda_B + \lambda_E, & T_e < t \leq T_E, \\ \lambda_b - \lambda_e - \lambda_B + \lambda_E, & t > T_E. \end{cases}$$

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We then have

$$
\begin{aligned}
m_{du} = & -\alpha_2 \operatorname{plim} \frac{1}{T}\sum_{t=1}^{T} d_t v_t \\
= & -\alpha_2 \left[\lambda_b \left(\lambda_b - \lambda_e\right)\left(\lambda_b - \lambda_e - \lambda_B + \lambda_E\right)\right. \\
& + \left(\lambda_B - \lambda_b\right)\left(1 - \lambda_e + \lambda_b\right)\left(1 - \lambda_e + \lambda_b - \lambda_B + \lambda_E\right) \\
& + \left(\lambda_e - \lambda_B\right)\left(1 - \lambda_e + \lambda_b\right)\left(-\lambda_e + \lambda_b - \lambda_B + \lambda_E\right) \\
& + \left(\lambda_E - \lambda_e\right)\left(\lambda_b - \lambda_e\right)\left(-1 - \lambda_e + \lambda_b - \lambda_B + \lambda_E\right) \\
& \left. + \left(1 - \lambda_E\right)\left(\lambda_b - \lambda_e\right)\left(\lambda_b - \lambda_e - \lambda_B + \lambda_E\right)\right] \\
= & -\alpha_2 \left[\left(\lambda_B - \lambda_b\right)\left(1 - \lambda_e + \lambda_b\right) + \left(\lambda_e - \lambda_b\right)\left(\lambda_E - \lambda_e\right)\right].
\end{aligned}
$$

Furthermore, we have that

$$
\begin{aligned}
\frac{1}{T}\sum_{t=1}^{T} d_t D_t = & \lambda_b \left(\lambda_b - \lambda_e\right)\left(\lambda_B - \lambda_E\right) \\
& + \left(\lambda_B - \lambda_b\right)\left(1 - \lambda_e + \lambda_b\right)\left(\lambda_B - \lambda_E\right) \\
& + \left(\lambda_e - \lambda_B\right)\left(1 - \lambda_e + \lambda_b\right)\left(1 - \lambda_E + \lambda_B\right) \\
& + \left(\lambda_E - \lambda_e\right)\left(\lambda_b - \lambda_e\right)\left(1 - \lambda_E + \lambda_B\right) \\
& + \left(1 - \lambda_E\right)\left(\lambda_b - \lambda_e\right)\left(\lambda_B - \lambda_E\right) \\
= & \lambda_e - \lambda_B + \left(\lambda_e - \lambda_b\right)\left(\lambda_B - \lambda_E\right),
\end{aligned}
$$

hence we find that

$$
m_{dp_{-1}} = \frac{\alpha_2}{1-\gamma}\left(\lambda_e - \lambda_B + \left(\lambda_e - \lambda_b\right)\left(\lambda_B - \lambda_E\right)\right).
$$

Together with

$$
\frac{1}{T}\sum_{t=1}^{T} D_t^2 = \left(1 - \lambda_E + \lambda_B\right)\left(\lambda_E - \lambda_B\right),
$$

we find

$$
\begin{aligned}
\frac{1}{T}\sum_{t=1}^{T} v_t D_t &= \frac{1}{T}\sum_{t=1}^{T} d_t D_t - \frac{1}{T}\sum_{t=1}^{T} D_t^2, \\
&= \lambda_e - \lambda_B + \left(1 - \lambda_E + \lambda_B + \lambda_e - \lambda_b\right)\left(\lambda_B - \lambda_E\right),
\end{aligned}
$$

and we therefore have

$$
m_{p_{-1}u} = -\frac{\alpha_2^2}{1-\gamma}\left[\lambda_e - \lambda_B + \left(1 - \lambda_E + \lambda_B + \lambda_e - \lambda_b\right)\left(\lambda_B - \lambda_E\right)\right].
$$

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Inspecting signs, we obviously have $m_{dd} > 0$. Furthermore, assuming the cartel effect $\alpha_2 > 0$ and given $T_b < T_B < T_e < T_E$ we find that $m_{du} < 0$ and $m_{dp_{-1}} > 0$ because

$$\begin{aligned}\lambda_e - \lambda_B + (\lambda_e - \lambda_b)(\lambda_B - \lambda_E) &< \lambda_E - \lambda_B + (\lambda_e - \lambda_b)(\lambda_B - \lambda_E)\\ &= (\lambda_E - \lambda_B)(1 + \lambda_b - \lambda_e)\\ &> 0.\end{aligned}$$

Also we find $m_{p_{-1}u} > 0$ because

$$\begin{aligned}&(\lambda_E - \lambda_B)(1 - \lambda_E + \lambda_B + \lambda_e - \lambda_b) - (\lambda_e - \lambda_B)\\ >&(\lambda_E - \lambda_B)(1 - \lambda_E + \lambda_B + \lambda_e - \lambda_B) - (\lambda_e - \lambda_B)\\ =&(1 - \lambda_E + \lambda_B)(\lambda_E - \lambda_e)\\ >&0,\end{aligned}$$

where the second line follows from the fact that $\lambda_B > \lambda_b$. Collecting terms we therefore find for Case 1 that

$$m_{dd}m_{p_{-1}u} - m_{du}m_{dp_{-1}} > 0.$$

Hence, we can write for the direction of the first three elements in the OLS inconsistency (A.4):

$$\begin{aligned}&\operatorname{plim}\hat{\gamma} > \gamma,\\ &\operatorname{plim}\hat{\beta} \lesseqgtr \beta, \quad \text{if } m_{xp_{-1}} \gtreqless 0,\\ &\operatorname{plim}\hat{\alpha} < \alpha,\end{aligned}$$

where the last inequality holds as prices are positive, hence $m_{p_{-1}} > 0$. Finally, the last element in the inconsistency is

$$\begin{aligned}\operatorname{plim}(\hat{\alpha}_2 - \alpha_2) &= \frac{-m_{dp_{-1}}m_{xx}m_{p_{-1}u} - m_{du}(\sigma^2_{xp_{-1}} - \sigma^2_x\sigma^2_p)}{\det(\Sigma_{ZZ})}\\ &= -\sigma^2_x\frac{m_{dp_{-1}}m_{p_{-1}u} - m_{du}\sigma^2_p(1 - \rho^2_{xp_{-1}})}{\det(\Sigma_{ZZ})}\\ &< 0,\end{aligned}$$

because $m_{dp_{-1}} > 0$, $m_{p_{-1}u} > 0$ and $m_{du} < 0$.

We find exactly the same qualitative results for the Cases 2, 3 and 4. Because Case 2 will turn out to be most important we briefly state the results for this case. In Case 2, i.e. $T_b < T_B$, $T_e > T_E$, we have for the measurement error:

$$v_t = \begin{cases}\lambda_b - \lambda_e - \lambda_B + \lambda_E, & t \leq T_b,\\ 1 - \lambda_e + \lambda_b - \lambda_B + \lambda_E, & T_b < t \leq T_B,\\ \lambda_b - \lambda_e - \lambda_B + \lambda_E, & T_B < t \leq T_E,\\ 1 - \lambda_e + \lambda_b - \lambda_B + \lambda_E, & T_E < t \leq T_e,\\ \lambda_b - \lambda_e - \lambda_B + \lambda_E, & t > T_e.\end{cases}$$

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We then have:

$$
\begin{aligned}
m_{du} &= -\alpha_2 \left(1 - \lambda_e + \lambda_b\right) \left(\lambda_B - \lambda_E + \lambda_e - \lambda_b\right), \\
m_{dp_{-1}} &= \frac{\alpha_2}{1-\gamma} \left(\lambda_E - \lambda_B\right) \left(1 - \lambda_e + \lambda_b\right), \\
m_{p_{-1}u} &= -\frac{\alpha_2^2}{1-\gamma} \left(\lambda_E - \lambda_B\right) \left(\lambda_b - \lambda_e + \lambda_E - \lambda_B\right).
\end{aligned}
$$

Inspecting signs, we find that $m_{du} < 0$ as

$$
\begin{aligned}
\left(1 - \lambda_e + \lambda_b\right) \left(\lambda_B - \lambda_E + \lambda_e - \lambda_b\right) &= \left(1 - \left(\lambda_e - \lambda_b\right)\right) \left(\lambda_e - \lambda_b - \left(\lambda_E - \lambda_B\right)\right) \\
&> 0,
\end{aligned}
$$

because in Case 2 $\lambda_e - \lambda_b > \lambda_E - \lambda_B$. Furthermore, it is obvious that $m_{dp_{-1}} > 0$ and $m_{p_{-1}u} > 0$. Collecting terms we therefore find the same qualitative results compared with Case 1.

For the original intercept α_1, we obtain the following. Defining

$$
c = \frac{m_{dd} m_{p_{-1}u} - m_{du} m_{dp_{-1}}}{\det\left(\Sigma_{ZZ}\right)},
$$

we can write

$$
\begin{aligned}
\operatorname{plim}\left(\frac{\hat{\alpha}}{1-\hat{\gamma}} - \frac{\alpha}{1-\gamma}\right) &= \frac{\operatorname{plim}\left(\hat{\alpha} - \alpha\right) + \frac{\alpha}{1-\gamma} \operatorname{plim}\left(\hat{\gamma} - \gamma\right)}{1 - \gamma - \operatorname{plim}\left(\hat{\gamma} - \gamma\right)} \\
&= \frac{-m_p m_{xx} c + \frac{\alpha}{1-\gamma} m_{xx} c}{1 - \gamma - m_{xx} c} \\
&= \left(\frac{m_{xx} c}{1 - \gamma - m_{xx} c}\right) \left(\frac{\alpha}{1-\gamma} - m_p\right) \\
&= 0.
\end{aligned}
$$

It can also be shown that

$$
\left(\lambda_E - \lambda_B\right) \frac{\alpha_2}{1-\gamma} - \left(\lambda_e - \lambda_b\right) \operatorname{plim} \frac{\hat{\alpha}_2}{1-\hat{\gamma}} = \begin{cases} 0, & T_b < T_B, T_e > T_E, \\ > 0, & \text{otherwise.} \end{cases} \tag{A.6}
$$

Therefore, given that $\operatorname{plim} \hat{\gamma} > \gamma$ it is easily seen that for Case 2

$$
\begin{aligned}
\operatorname{plim} \hat{\alpha}_1 &= \operatorname{plim} \hat{\alpha} - \left(\lambda_e - \lambda_b\right) \operatorname{plim} \hat{\alpha}_2 \\
&= \alpha_1 \frac{\operatorname{plim}\left(1 - \hat{\gamma}\right)}{1-\gamma} \\
&< \alpha_1.
\end{aligned}
$$

Numerical simulations confirm that it holds for the other cases as well. □

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Proof of Proposition 4.

(i) Recall that

$$t \leq T_b : \widehat{bfp}_t = p_t,$$
$$t > T_b : \widehat{bfp}_t = \hat{\gamma}\widehat{bfp}_{t-1} + \hat{\beta}x_t + \hat{\alpha}_1,$$

and

$$bfp_t = \gamma bfp_{t-1} + \beta x_t + \alpha_1 + \varepsilon_t,$$

where in terms of the parameter vector $\theta = (\gamma, \beta, \alpha, \alpha_1)'$, we have $\hat{\alpha}_1 = \hat{\alpha} - (\lambda_e - \lambda_b)\hat{\alpha}_2$ and $\alpha_1 = \alpha - (\lambda_E - \lambda_B)\alpha_2$. We have for the average estimated but-for price

$$\begin{aligned} \text{plim}\,\frac{1}{T_e - T_b}\sum_{t=T_b+1}^{T_e} \widehat{bfp}_t &= \text{plim}\,\frac{1}{T_e - T_b}\sum_{t=T_b+1}^{T_e} bfp_t + \text{plim}\,\frac{1}{T_e - T_b}\sum_{t=T_b+1}^{T_e}\left(\widehat{bfp}_t - bfp_t\right) \\ &= \text{plim}\,\frac{1}{T_E - T_B}\sum_{t=T_B+1}^{T_E} bfp_t + \text{plim}\,\frac{1}{T_e - T_b}\sum_{t=T_b+1}^{T_e}\left(\widehat{bfp}_t - bfp_t\right). \end{aligned}$$

We will analyze the limiting behavior of the second term in more detail. In period $T_b + 1$ we can write for the prediction error

$$\begin{aligned} v_{T_b+1} &= \widehat{bfp}_{T_b+1} - bfp_{T_b+1} \\ &= (\hat{\gamma} - \gamma)\, bfp_{T_b} + \hat{\gamma}\,(p_{T_b} - bfp_{T_b}) + (\hat{\beta} - \beta)x_{T_b+1} + \hat{\alpha} - \alpha - \varepsilon_{T_b+1}. \end{aligned}$$

Note that standard asymptotic theory for method of moments estimators gives the following large sample distribution of the OLS estimator

$$\sqrt{T}\left(\hat{\theta} - \theta - \theta^*\right) \xrightarrow{d} \mathcal{N}(0, V),$$

where $\theta^* = \Sigma_{ZZ}^{-1}\Sigma_{Zu}$ is the inconsistency. Therefore, we can write $\hat{\theta} - \theta = \theta^* + O_P(T^{-1/2})$. The implied inconsistency in α_1 is

$$\alpha_1^* = \alpha^* - (\lambda_e - \lambda_b)\alpha_2^* + (\lambda_E - \lambda_B - (\lambda_e - \lambda_b))\alpha_2.$$

Also note that we have:

$$bfp_{T_b} = \begin{cases} p_{T_b}, & T_b \leq T_B, \\ < p_{T_b}, & T_b > T_B, \end{cases}$$

so that:

$$\hat{\gamma}\,(p_{T_b} - bfp_{T_b}) = \begin{cases} 0, & T_b \leq T_B, \\ (\gamma + \gamma^*)\,(p_{T_b} - bfp_{T_b}) + O_P(T^{-1/2}), & T_b > T_B. \end{cases}$$

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Therefore we can write:

$$v_{T_b+1} = \begin{cases} \gamma^* bfp_{T_b} + \beta^* x_{T_b+1} + \alpha_1^* - \varepsilon_{T_b+1} + O_P(T^{-1/2}), & T_b \leq T_B, \\ \gamma^* bfp_{T_b} + \beta^* x_{T_b+1} + \alpha_1^* - \varepsilon_{T_b+1} + O_P(1) & T_b > T_B. \end{cases}$$

In period $T_b + 2$ we have for the difference in estimated and true but-for prices

$$\begin{aligned} \widehat{bfp}_{T_b+2} - bfp_{T_b+2} &= \hat{\gamma}\widehat{bfp}_{T_b+1} - \gamma bfp_{T_b+1} + (\hat{\beta} - \beta)x_{T_b+2} + \hat{\alpha}_1 - \alpha_1 - \varepsilon_{T_b+2} \\ &= \hat{\gamma} v_{T_b+1} + (\hat{\gamma} - \gamma)\, bfp_{T_b+1} + (\hat{\beta} - \beta)x_{T_b+2} + \hat{\alpha}_1 - \alpha_1 - \varepsilon_{T_b+2} \\ &= \hat{\gamma} v_{T_b+1} + v_{T_b+2}. \end{aligned}$$

Regarding the prediction error v_{T_b+2} we find

$$v_{T_b+2} = \gamma^* bfp_{T_b+1} + \beta^* x_{T_b+2} + \alpha_1^* - \varepsilon_{T_b+2} + O_P(T^{-1/2}),$$

irrespective of the precise timing of T_b and T_B. In general we find

$$\begin{aligned} \widehat{bfp}_{T_b+s} - bfp_{T_b+s} &= \hat{\gamma}^{s-1} v_{T_b+1} + \hat{\gamma}^{s-2} v_{T_b+2} + \ldots + \hat{\gamma} v_{T_b+s-1} + v_{T_b+s} \\ &= \sum_{j=0}^{s-1} \hat{\gamma}^j v_{T_b+s-j} \\ &= \sum_{j=0}^{s-1} (\gamma + \gamma^*)^j\, v_{T_b+s-j} + O_P(T^{-1/2}), \end{aligned}$$

where

$$v_{T_b+s-j} = \gamma^* bfp_{T_b+s-j-1} + \beta^* x_{T_b+s-j} + \alpha_1^* - \varepsilon_{T_b+s-j} + O_P(T^{-1/2}), \quad j = 0, \ldots, s-2.$$

By repeated substitution we can write

$$bfp_t = \frac{\alpha_1}{1-\gamma} + \beta \sum_{i=0}^{\infty} \gamma^i x_{t-i} + \sum_{i=0}^{\infty} \gamma^i \varepsilon_{t-i},$$

hence

$$E\left[bfp_t\right] = \frac{\alpha_1}{1-\gamma},$$

because $E\left[x_t\right] = 0$ and $E\left[\varepsilon_t\right] = 0$. Defining

$$c = \frac{m_{dd} m_{p_{-1}u} - m_{du} m_{dp_{-1}}}{\det\left(\Sigma_{ZZ}\right)},$$

we can write

$$\begin{aligned} \operatorname{plim}\left(\frac{\hat{\alpha}}{1-\hat{\gamma}} - \frac{\alpha}{1-\gamma}\right) &= \frac{\operatorname{plim}\left(\hat{\alpha} - \alpha\right) + \alpha \operatorname{plim}\left(\hat{\gamma} - \gamma\right)/(1-\gamma)}{1 - \gamma - \operatorname{plim}\left(\hat{\gamma} - \gamma\right)} \\ &= \frac{-m_p m_{xx} c + m_{xx} c\alpha/(1-\gamma)}{1 - \gamma - m_{xx} c} \\ &= 0, \end{aligned}$$

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because $m_p = \alpha/(1-\gamma)$.

Using $\gamma^* = m_{xx}c$ and $\alpha^* = -m_p m_{xx} c$ once more we get

$$\begin{aligned}
E\left[v_{T_b+s-j}\right] &= E\left[\gamma^* bfp_{T_b+s-j-1} + \beta^* x_{T_b+s-j} + \alpha_1^* - \varepsilon_{T_b+s-j}\right] + O(T^{-1}) \\
&= m_{xx}c\left(\frac{\alpha}{1-\gamma} - (\lambda_E - \lambda_B)\frac{\alpha_2}{1-\gamma}\right) - m_p m_{xx} c \\
&\quad - ((\lambda_e - \lambda_b)\,\mathrm{plim}\,\hat{\alpha}_2 - (\lambda_E - \lambda_B)\,\alpha_2) + O(T^{-1}) \\
&= -\,\mathrm{plim}\,(\hat{\gamma} - \gamma)\,(\lambda_E - \lambda_B)\,\frac{\alpha_2}{1-\gamma} - (\lambda_e - \lambda_b)\,\mathrm{plim}\,\hat{\alpha}_2 \\
&\quad + (\lambda_E - \lambda_B)\,\alpha_2 + O(T^{-1}) \\
&= (\lambda_E - \lambda_B)\,\frac{\alpha_2}{1-\gamma}(1 - \mathrm{plim}\,\hat{\gamma}) - (\lambda_e - \lambda_b)\,\mathrm{plim}\,\hat{\alpha}_2.
\end{aligned}$$

From (A.6) we see that:

$$\mathrm{plim}\,\hat{\alpha}_2 = \begin{cases} \dfrac{\lambda_E - \lambda_B}{\lambda_e - \lambda_b}\dfrac{\alpha_2}{1-\gamma}(1 - \mathrm{plim}\,\hat{\gamma}), & T_b < T_B, T_e > T_E, \\ < \dfrac{\lambda_E - \lambda_B}{\lambda_e - \lambda_b}\dfrac{\alpha_2}{1-\gamma}(1 - \mathrm{plim}\,\hat{\gamma}), & \text{otherwise.} \end{cases} \tag{A.7}$$

Using this we find:

$$E\left[v_{T_b+s-j}\right] = \begin{cases} O(T^{-1}), & T_b < T_B, T_e > T_E, \\ O(1) > 0, & \text{otherwise.} \end{cases}$$

The difference in estimated and true but-for price is

$$\widehat{bfp}_{T_b+s} - bfp_{T_b+s} = \sum_{j=0}^{s-1} (\gamma + \gamma^*)^j \, v_{T_b+s-j} + O_P(T^{-1/2}),$$

with:

$$E\left[\sum_{j=0}^{s-1} (\gamma + \gamma^*)^j \, v_{T_b+s-j}\right] = \begin{cases} O(T^{-1}), & T_b < T_B, T_e > T_E, \\ O(1) > 0, & \text{otherwise.} \end{cases}$$

Hence

$$\begin{aligned}
&\mathrm{plim}\,\frac{1}{T_e - T_b}\sum_{s=1}^{T_e - T_b}\left(\widehat{bfp}_{T_b+s} - bfp_{T_b+s}\right) \\
&= \mathrm{plim}\,\frac{1}{T_e - T_b}\sum_{s=1}^{T_e - T_b}\sum_{j=0}^{s-1} (\gamma + \gamma^*)^j \, v_{T_b+s-j} = \begin{cases} 0, & T_b < T_B, T_e > T_E, \\ > 0, & \text{otherwise,} \end{cases}
\end{aligned}$$

which completes the proof of (i).

(ii) Noting that $bfp_{T_B} = p_{T_B}$ we have for the overcharge

$$O_{T_B+t} = (p_{T_B+t} - bfp_{T_B+t})\,D_t = \frac{1-\gamma^t}{1-\gamma}\alpha_2 D_t,$$

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which for $t \to \infty$ simplifies to $\alpha_2/(1-\gamma)$. Therefore, we have for the average effective overcharge

$$\begin{aligned}\operatorname{plim} \bar{O} &= \operatorname{plim} \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} (p_t - bfp_t) \\ &= \lim \frac{1}{T_E - T_B} \sum_{t=T_B+1}^{T_E} \frac{1-\gamma^t}{1-\gamma} \alpha_2 \\ &= \frac{\alpha_2}{1-\gamma}.\end{aligned}$$

We have for the average estimated overcharge

$$\begin{aligned}\operatorname{plim} \overline{O}_2 &= \operatorname{plim} \frac{1}{T_e - T_b} \sum_{t=T_b+1}^{T_e} \left(p_t - \widehat{bfp}_t\right) \\ &= \operatorname{plim} \frac{1}{T_e - T_b} \sum_{t=T_b+1}^{T_e} (p_t - bfp_t) - \operatorname{plim} \frac{1}{T_e - T_b} \sum_{t=T_b+1}^{T_e} \left(\widehat{bfp}_t - bfp_t\right).\end{aligned}$$

Using Proposition 4 (i) we find that

$$\begin{aligned}\operatorname{plim} \overline{O}_2 &= \operatorname{plim} \frac{1}{T_e - T_b} \sum_{t=T_b+1}^{T_e} (p_t - bfp_t), \quad T_b < T_B, T_e > T_E, \\ &< \operatorname{plim} \frac{1}{T_e - T_b} \sum_{t=T_b+1}^{T_e} (p_t - bfp_t), \quad \text{otherwise.}\end{aligned}$$

Combining this with the fact that

$$\begin{aligned}\operatorname{plim} \frac{1}{T_e - T_b} \sum_{t=T_b+1}^{T_e} (p_t - bfp_t) &= \frac{\alpha_2}{1-\gamma} \quad T_b > T_B, T_e < T_E, \\ &< \frac{\alpha_2}{1-\gamma}, \quad \text{otherwise,}\end{aligned}$$

completes the proof. □

Proof of Theorem 3. We have for the effective damage

$$\begin{aligned}\operatorname{plim} \frac{1}{T} CD &= \operatorname{plim} \frac{1}{T} \sum_{t=T_B+1}^{T_E} (p_t - bfp_t)\, Q_t \\ &= \operatorname{plim} \frac{1}{T} \sum_{t=T_B+1}^{T_E} \frac{1-\gamma^t}{1-\gamma} \alpha_2 Q_t \\ &= \lim \frac{T_E - T_B}{T} \alpha_2 Q_C \lim \frac{1}{T} \sum_{t=T_B+1}^{T_E} \frac{1-\gamma^t}{1-\gamma} \\ &= \frac{\alpha_2}{1-\gamma} (\lambda_E - \lambda_B)\, Q_C.\end{aligned}$$

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We have for the estimated damage

$$\begin{aligned}
\text{plim}\,\frac{1}{T}\widehat{CD}_2 &= \text{plim}\,\frac{1}{T}\sum_{t=T_b+1}^{T_e}\left(p_t - \widehat{bfp}_t\right)Q_t \\
&= \text{plim}\,\frac{1}{T}\sum_{t=T_b+1}^{T_e}\left(p_t - bfp_t\right)Q_t + \text{plim}\,\frac{1}{T}\sum_{t=T_b+1}^{T_e}\left(bfp_t - \widehat{bfp}_t\right)Q_t \\
&= \text{plim}\,\frac{1}{T}CD - \text{plim}\,\frac{1}{T}\sum_{t=T_b+1}^{T_e}\left(\widehat{bfp}_t - bfp_t\right)Q_t.
\end{aligned}$$

We will analyze the limiting behavior of the second term in more detail under Assumption 1. Proposition 4 (i) implies that

$$\left(\widehat{bfp}_{T_b+s} - bfp_{T_b+s}\right)Q_{T_b+s} = Q_{T_b+s}\sum_{j=0}^{s-1}\left(\gamma+\gamma^*\right)^j v_{T_b+s-j} + O_P(T^{-1/2}),$$

where

$$v_{T_b+s-j} = \gamma^* bfp_{T_b+s-j-1} + \beta^* x_{T_b+s-j} + \alpha_1^* - \varepsilon_{T_b+s-j} + O_P(T^{-1/2}), \quad j = 0,\ldots,s-2.$$

We note that

$$bfp_{t-1} = \frac{\alpha_1}{1-\gamma} + \beta\sum_{s=0}^{\infty}\gamma^s x_{t-1-s} + \sum_{s=0}^{\infty}\gamma^s\varepsilon_{t-1-s}.$$

Due to $m_x = 0$, we can define the linear projection coefficients of a regression of bfp_{t-1} on a constant and x_t as m_p and $\frac{m_{xp_{-1}}}{m_{xx}}$ respectively. Therefore, we can write

$$\begin{aligned}
bfp_{t-1} &= m_p + \frac{m_{xp_{-1}}}{m_{xx}}x_t + \eta_t, \\
\eta_t &= \beta\sum_{s=0}^{\infty}\gamma^s x_{t-1-s} - \frac{m_{xp_{-1}}}{m_{xx}}x_t + \sum_{s=0}^{\infty}\gamma^s\varepsilon_{t-1-s},
\end{aligned}$$

where by construction $E\left[\eta_t x_t\right] = 0$. This leads to

$$E\left[bfp_{T_b+s-j-1}Q_{T_b+s}\right] = \frac{m_{xp_{-1}}}{m_{xx}}E\left[x_{T_b+s-j}Q_{T_b+s}\right] + \frac{\alpha_1}{1-\gamma}E\left[Q_{T_b+s}\right],$$

where we assumed that the projection error η_t is uncorrelated with Q_{T_b+s}.

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Using $\gamma^* = m_{xx}c$, $\beta^* = -m_{xp_{-1}}c$ and $\alpha^* = -m_p m_{xx}c$ we get

$$\begin{aligned}
E\left[v_{T_b+s-j}Q_{T_b+s}\right] &= E\left[\left(\gamma^* bfp_{T_b+s-j-1} + \beta^* x_{T_b+s-j} + \alpha_1^* - \varepsilon_{T_b+s-j}\right)Q_{T_b+s}\right] + O(T^{-1}) \\
&= m_{xx}c\frac{m_{xp_{-1}}}{m_{xx}}E\left[x_{T_b+s-j}Q_{T_b+s}\right] \\
&\quad + m_{xx}c\left(\frac{\alpha}{1-\gamma} - \left(\lambda_E - \lambda_B\right)\frac{\alpha_2}{1-\gamma}\right)E\left[Q_{T_b+s}\right] \\
&\quad - m_{xp_{-1}}cE\left[x_{T_b+s-j}Q_{T_b+s}\right] \\
&\quad - m_p m_{xx}cE\left[Q_{T_b+s}\right] \\
&\quad - \left(\left(\lambda_e - \lambda_b\right)\operatorname{plim}\hat{\alpha}_2 - \left(\lambda_E - \lambda_B\right)\alpha_2\right)E\left[Q_{T_b+s}\right] \\
&\quad - E\left[\varepsilon_{T_b+s-j}Q_{T_b+s}\right] + O(T^{-1}).
\end{aligned}$$

Collecting terms, this leads to

$$\begin{aligned}
E\left[v_{T_b+s-j}Q_{T_b+s}\right] = E\left[Q_{T_b+s}\right]&\left\{\left(\lambda_E - \lambda_B\right)\frac{\alpha_2}{1-\gamma}(1 - \operatorname{plim}\hat{\gamma}) - \left(\lambda_e - \lambda_b\right)\operatorname{plim}\hat{\alpha}_2\right\} \\
&- E\left[\varepsilon_{T_b+s-j}Q_{T_b+s}\right] + O(T^{-1}).
\end{aligned}$$

Using (A.7) we find:

$$E\left[v_{T_b+s-j}Q_{T_b+s}\right] = \begin{cases} -E\left[\varepsilon_{T_b+s-j}Q_{T_b+s}\right] + O(T^{-1}), & T_b < T_B, T_e > T_E, \\ -E\left[\varepsilon_{T_b+s-j}Q_{T_b+s}\right] + O(1), & \text{otherwise,} \end{cases}$$

where the $O(1)$ remainder term is positive. The difference in estimated and true damage per period is

$$\left(\widehat{bfp}_{T_b+s} - bfp_{T_b+s}\right)Q_{T_b+s} = \sum_{j=0}^{s-1}\left(\gamma + \gamma^*\right)^j v_{T_b+s-j}Q_{T_b+s} + O_P(T^{-1/2}),$$

hence

$$\begin{aligned}
&\operatorname{plim}\frac{1}{T_e - T_b}\sum_{s=1}^{T_e-T_b}\left(\widehat{bfp}_{T_b+s} - bfp_{T_b+s}\right)Q_{T_b+s} \\
=&\operatorname{plim}\frac{1}{T_e - T_b}\sum_{s=1}^{T_e-T_b}\sum_{j=0}^{s-1}\left(\gamma + \gamma^*\right)^j v_{T_b+s-j}Q_{T_b+s} \\
=&-\sum_{j=0}^{s-1}\left(\gamma + \gamma^*\right)^j E\left[\varepsilon_{T_b+s-j}Q_{T_b+s}\right] \\
\geq&0,
\end{aligned}$$

when $T_b < T_B, T_e > T_E$ and under Assumption 2. In the other cases a strict inequality sign holds. This shows that:

$$\begin{aligned}
\operatorname{plim}\frac{1}{T}\widehat{CD}_2 &\leq \operatorname{plim}\frac{1}{T}CD, \quad T_b < T_B, T_e > T_E \\
&< \operatorname{plim}\frac{1}{T}CD, \quad \text{otherwise,}
\end{aligned}$$

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which completes the proof. □

Appendix B: Simulation Results

For the Monte Carlo study, data have been generated according to DGP (3) with $\varepsilon_t \sim$ i.i.d. $N(0, \sigma_\varepsilon^2)$. Explanatory variables are lagged prices p_{t-1} and, for simplicity, a single cost factor x_t. The cartel dummy is defined as in (2) and set at $T_B = \frac{1}{3}T$ and $T_E = \frac{2}{3}T$. The explanatory variable x_t follows an AR(1) model

$$x_t = \rho x_{t-1} + v_t, \tag{A.8}$$

where $v_t \sim$ i.i.d. $N(0, \sigma_v^2)$ independent of ε_t, i.e. the cost factor is assumed to be strictly exogenous.

In order to investigate the actual size of various structural break inference procedures, data have been generated under the null hypothesis $H_0 : \alpha_2 = 0$. All experiments have a sample of $T = 100$ observations and the number of replications is 5,000. Without loss of generality we set $\alpha_1 = 100(1 - \gamma)$, so that the average price level in the simulations is 100. We normalized with respect to the variance of the disturbance term σ_ε^2. We furthermore choose $\gamma \in \{0.1, 0.5, 0.9\}$ and $\rho \in \{0.1, 0.5, 0.9\}$. These values roughly correspond to a low, intermediate and high degree of serial correlation in the time series p_t and x_t.

To facilitate the comparison of simulation results across experiments, some important design parameters are held fixed. We always set β such that the long-run effect of x on p is unity, i.e. we specify $\beta = 1 - \gamma$. Furthermore, σ_ε^2 is chosen such that the signal-to-noise ratio of the model, defined as

$$\text{SNR} = \frac{\text{Var}(p_t - \varepsilon_t)}{\text{Var}(\varepsilon_t)}, \tag{A.9}$$

does not change between experiments. Assuming $\sigma_\varepsilon^2 = 1$ for DGP (3), Kiviet (1995) derives the following relation between σ_v^2 and SNR and other model parameters

$$\sigma_v^2 = \frac{1}{\beta^2} \left[\text{SNR} - \frac{\gamma^2}{1 - \gamma^2} \right] \frac{(1 - \gamma^2)(1 - \rho^2)(1 - \gamma\rho)}{1 + \gamma\rho}. \tag{A.10}$$

We choose SNR $= 9$ across experiments corresponding with a population R^2 of 0.9.

The correct dynamic specification (3) has been estimated in all experiments, hence OLS estimators are consistent, and analyze actual rejection probabilities of sup F-tests and double maximum (UD and WD) tests proposed by Bai and Perron (1998, 2003, 2006).[2] The nominal significance level in the simulations is 5% always. For the sup F-tests, the null hypothesis is no break versus k breaks where we experimented with $k = \{1, 2, 3\}$. The

[2] We used EViews 9 for all calculations.

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finite sample properties of the various test procedures have been investigated for trimming parameter $\mu = h/T = 0.15$, which implies that a minimum of 15 observations is used in any partition of the data, given that $T = 100$.[3]

The bootstrap version of the various testing procedures uses a standard non-parametric resampling scheme. First, we obtained the OLS estimator allowing for breaks. Second, a random sample is taken from the empirical distribution of the OLS residuals. Third, the bootstrapped dependent variable is calculated according to equation (3). In the bootstrap scheme, we kept the values of exogenous regressors as before. For the lagged dependent variable regressor, the first observation on the dependent variable is kept also as before and pseudo values for the remaining observations are constructed iteratively. Fourth, we estimated the model and calculated the various test statistics from the resampled data. Repeating steps 2 to 4 of the resampling scheme B times, together with the calculated test statistic on the original data $(B + 1)$ generated realizations of the test statistic. A size-corrected test was constructed with these $(B + 1)$ realizations by using the appropriate quantile of the bootstrap distribution as critical value. The number of bootstrap replications $B = 199$.

Table B.1 shows actual rejection probabilities of asymptotic and bootstrap sup F-tests and double maximum tests. Autoregressive dynamics are varied in both p_t and x_t to analyze their relevance for the accuracy of asymptotic approximations. Asymptotic tests performed well under the null hypothesis when there is only small or moderate autoregressive dynamics in the dependent variable, i.e. $\gamma \in \{0.1, 0.5\}$. Rejection frequencies do not exceed 0.10, which is reasonably close to the nominal level of 0.05. However, these tests become oversized when $\gamma = 0.9$. In this case, actual rejection frequencies are also increasing with the persistence in the regressor x. In addition, size distortions of sup F-tests increase with the number of breaks specified under H_1. In general, size distortions of asymptotic tests are largest when both p and x are highly serially correlated ($\gamma = \rho = 0.9$), corroborating the simulation results for univariate models.[4] In contrast, bootstrap tests are always size correct, irrespective of the true value of the autoregressive dynamics. Therefore, in practice one should favour the bootstrap version of the break tests always.

We also analyzed power of the various asymptotic and bootstrap break tests. Data have been generated as before, but now under the alternative hypothesis $H_1 : \alpha_2 \neq 0$. A 20% overcharge, i.e. $\alpha_2 = 0.2\alpha_1$, is in line with findings reported in Connor and Lande (2008) and the European Commission's 2013 *Practical Guide*. Table B.2 shows size-corrected power of asymptotic and bootstrap sup F-tests and double maximum tests. The power

[3]We obtained similar size and power properties result for $\mu = 0.05$.

[4]Diebold and Chen (1996), O'Reilly and Whelan (2005). Unreported simulation results show the same pattern for asymptotic sequential sup F tests.

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Table B.1: Size of nominal 5% $\sup F$- and double max tests

γ	ρ	$\sup F(1)$	$\sup F(2)$	$\sup F(3)$	UD max	WD max
			asymptotic tests			
0.1	0.1	0.054	0.048	0.046	0.054	0.049
0.1	0.5	0.052	0.051	0.050	0.053	0.054
0.1	0.9	0.059	0.060	0.061	0.060	0.061
0.5	0.1	0.057	0.054	0.056	0.058	0.059
0.5	0.5	0.058	0.059	0.063	0.060	0.064
0.5	0.9	0.067	0.074	0.091	0.074	0.092
0.9	0.1	0.136	0.211	0.288	0.187	0.288
0.9	0.5	0.140	0.232	0.324	0.204	0.318
0.9	0.9	0.195	0.322	0.458	0.301	0.484
			bootstrap tests			
0.1	0.1	0.056	0.047	0.041	0.051	0.041
0.1	0.5	0.054	0.048	0.046	0.051	0.043
0.1	0.9	0.053	0.050	0.045	0.053	0.042
0.5	0.1	0.053	0.048	0.042	0.050	0.042
0.5	0.5	0.053	0.054	0.049	0.053	0.047
0.5	0.9	0.052	0.050	0.047	0.050	0.040
0.9	0.1	0.056	0.052	0.051	0.055	0.044
0.9	0.5	0.055	0.053	0.051	0.054	0.045
0.9	0.9	0.057	0.057	0.057	0.056	0.044

Note: $T = 100, \mu = 0.15, \alpha_2 = 0$.

properties of all tests are satisfactory, but power decreases when persistence in the data becomes larger, as expected. Generally, for this DGP with two structural breaks the power of the sup $F(1)$ test for a single break is lower than the power of tests for multiple breaks. This corroborates the simulation evidence of Bai and Perron (2006), who find that an underspecification of the number of breaks leads to a power loss in finite samples.

To analyze the sensitivity of these results, in Table B.3 we report power for a lower bound 10% overcharge, i.e. $\alpha_2 = 0.1\alpha_1$. Power is still satisfactory for small to moderate values of γ, but in case of highly persistent time series a deterioration of rejection frequencies can be seen compared to the case of a 20% overcharge. This is as expected, because with a relatively low overcharge it is more difficult to distinguish between serial correlation and structural change.

Finally, we analyze the finite sample accuracy of OLS inference conditional on the estimated break dates. We assume throughout that the correct number of breaks has been

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Table B.2: Size-corrected power, 20% overcharge

γ	ρ	$\sup F(1)$	$\sup F(2)$	$\sup F(3)$	UD max	WD max
			asymptotic tests			
0.1	0.1	0.544	1.000	1.000	1.000	1.000
0.1	0.5	0.484	1.000	1.000	1.000	1.000
0.1	0.9	0.296	1.000	1.000	1.000	1.000
0.5	0.1	0.699	1.000	1.000	1.000	1.000
0.5	0.5	0.662	1.000	1.000	1.000	1.000
0.5	0.9	0.537	1.000	0.999	1.000	1.000
0.9	0.1	0.647	0.999	0.998	0.997	0.998
0.9	0.5	0.628	0.997	0.996	0.995	0.995
0.9	0.9	0.473	0.947	0.956	0.951	0.969
			bootstrap tests			
0.1	0.1	0.419	1.000	1.000	1.000	1.000
0.1	0.5	0.347	1.000	1.000	1.000	1.000
0.1	0.9	0.208	1.000	1.000	1.000	1.000
0.5	0.1	0.600	1.000	1.000	1.000	1.000
0.5	0.5	0.566	1.000	1.000	1.000	1.000
0.5	0.9	0.404	1.000	0.999	1.000	1.000
0.9	0.1	0.546	0.999	0.993	0.991	0.991
0.9	0.5	0.509	0.994	0.985	0.984	0.981
0.9	0.9	0.364	0.928	0.924	0.904	0.916

Note: $T = 100, \mu = 0.15, \alpha_2 = 0.2\alpha_1$. Nominal size is 0.05.

estimated, which in the experiments is always equal to two breaks. In each replication of an experiment, we base the definition of the cartel dummy on the estimated break dates. In Tables B.4 and B.5 we report the bias and Monte Carlo standard deviation of the OLS estimator of the cartel effect α_2. Furthermore, we show the actual rejection frequency of the corresponding nominal 5% t-test of $H_0 : \alpha_2 = \alpha_{2,0}$ with $\alpha_{2,0}$ the true value.

In case of a 20% overcharge (Table B.4) the bias of the coefficient estimator is of moderate magnitude and is small compared to the Monte Carlo standard deviation. Note that the bias of the OLS estimator of α_2 can have either sign depending on the parameters of the model. Actual rejection frequencies indicate that OLS inference is somewhat conservative for moderate values of γ, while overrejection is seen for experiments with high γ. In general, rejection frequencies are reasonably close to the nominal level of 0.05. Regarding a 10% overcharge (Table B.5), finite sample accuracy deteriorates as expected. We conclude that, at least in this set of experiments, the finite sample properties of OLS inference are

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Table B.3: Size-corrected power, 10% overcharge

γ	ρ	sup $F(1)$	sup $F(2)$	sup $F(3)$	UD max	WD max
			asymptotic tests			
0.1	0.1	0.990	1.000	1.000	1.000	1.000
0.1	0.5	0.947	1.000	1.000	1.000	1.000
0.1	0.9	0.467	1.000	1.000	1.000	1.000
0.5	0.1	0.909	1.000	1.000	1.000	1.000
0.5	0.5	0.847	1.000	1.000	1.000	1.000
0.5	0.9	0.486	0.999	0.999	1.000	1.000
0.9	0.1	0.100	0.701	0.627	0.488	0.592
0.9	0.5	0.095	0.659	0.587	0.459	0.544
0.9	0.9	0.077	0.498	0.421	0.315	0.414
			bootstrap tests			
0.1	0.1	0.950	1.000	1.000	1.000	1.000
0.1	0.5	0.837	1.000	1.000	1.000	1.000
0.1	0.9	0.355	1.000	1.000	1.000	1.000
0.5	0.1	0.827	1.000	1.000	1.000	1.000
0.5	0.5	0.751	1.000	1.000	1.000	1.000
0.5	0.9	0.366	0.998	0.999	1.000	1.000
0.9	0.1	0.066	0.573	0.476	0.423	0.287
0.9	0.5	0.055	0.526	0.345	0.297	0.184
0.9	0.9	0.043	0.356	0.393	0.346	0.223

Note: $T = 100, \mu = 0.15, \alpha_2 = 0.1\alpha_1$. Nominal size is 0.05.

satisfactory.

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Table B.4: OLS inference, 20% overcharge

γ	ρ	α_2	bias $\hat{\alpha}_2$	sd $\hat{\alpha}_2$	rf $t_{\hat{\alpha}_2}$
0.1	0.1	18	−0.329	0.981	0.027
0.1	0.5	18	−0.298	1.047	0.026
0.1	0.9	18	−0.165	1.258	0.010
0.5	0.1	10	−0.099	0.645	0.031
0.5	0.5	10	−0.080	0.673	0.029
0.5	0.9	10	−0.054	1.128	0.021
0.9	0.1	2	0.143	0.369	0.080
0.9	0.5	2	0.122	0.452	0.089
0.9	0.9	2	−0.088	0.907	0.157

Note: $T = 100, \mu = 0.15, \alpha_2 = 0.2\alpha_1$. Nominal size is 0.05.

sd is Monte Carlo standard deviation, rf is rejection frequency.

Table B.5: OLS inference, 10% overcharge

γ	ρ	α_2	bias $\hat{\alpha}_2$	sd $\hat{\alpha}_2$	rf $t_{\hat{\alpha}_2}$
0.1	0.1	9	−0.217	0.446	0.052
0.1	0.5	9	−0.206	0.487	0.051
0.1	0.9	9	−0.183	0.984	0.042
0.5	0.1	5	−0.082	0.384	0.052
0.5	0.5	5	−0.068	0.408	0.049
0.5	0.9	5	−0.075	0.865	0.059
0.9	0.1	1	0.277	0.371	0.146
0.9	0.5	1	0.270	0.410	0.155
0.9	0.9	1	0.238	0.559	0.191

Note: $T = 100, \mu = 0.15, \alpha_2 = 0.1\alpha_1$. Nominal size is 0.05.

sd is Monte Carlo standard deviation, rf is rejection frequency.

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